NAME ____

Do all problems. No calculators. Points per problem listed on the back page.

Problem 1: Solving a linear equation

Given matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & 3 & 2 & 1 \end{bmatrix}$ and vector $y = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

(a) Solve Ax = y (if the equation is consistent) and write the general solution x in (vector) parametric form.

(b) Write a basis for the null space of A. **Basis = ____________________**

(c) What is the dimension of the range of A? **Dimension = _____**

(d) Is y in the span of the row vectors of A? **Yes? No?**

Problem 2: Conclusions from echelon form.

In each case, we start with a matrix A and vector and tell what one will get by reducing the augmented matrix of the system Ax = y to echelon form. Answer the questions in each case using this information.

A	y	Echelon form of augmented matrix of Ax = y.
$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 2 \\ 4 & 4 & 4 & 1 \end{bmatrix}$	$y = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Write the general solution for $Ax = y$ in (vector) parametric form
Solution:

What is the dimension of the null space of A? **Dimension = ____**

Write down a basis for the null space of A. **Basis = __________**

Is y in the range of A? **Yes? No?**

What is the dimension of the range of A? **Dimension =** ______

Write down a basis of the range of A. **Basis = __________**

Are the columns of A independent? **Yes? No?**

B	z	Echelon form of augmented matrix of Bx = z.
B = $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix}$	z = $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Write the general solution for Bx = z in (vector) parametric form

Solution:

What is the dimension of the null space of B? **Dimension = ______**

Write down a basis for the null space of B. **Basis = __________**

Is y in the range of B? **Yes? No?**

What is the dimension of the range of B? **Dimension = ______**

Write down a basis of the range of B. **Basis = __________**

Are the columns of B independent? **Yes? No?**

C	Reduced row echelon form of C.
C = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Is C invertible? **Yes? No?**

Are the columns of C independent? **Yes? No?**

Write down a basis for the null space of C.

Problem 3: Compute AB

Compute the stated matrix products (if defined) for these matrices.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 3 \end{bmatrix},\ C = \begin{bmatrix} -2 \\ 1 \\ 3 \\ 1 \end{bmatrix},\ D = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Compute each of the following matrix products or other matrices (if defined):

A^{-1}	C^{-1}
AB	BA
CD	BC
CD^T	C^TD

Problem 4: Transpose and product

Suppose M is a 4 x 3 matrix whose columns M1, M2, M3 are orthogonal and have lengths |M1| = 2, |M2| = 3, |M3| = 4. Tell what are the entries in the product M^TM, as much and as precisely as possible from this information.

M^TM =

Problem 5: Find the eigenvalues and eigenvectors

Find the eigenvalues and eigenvectors of matrix M = $\begin{bmatrix} 0 & -2 \\ 2 & -4 \end{bmatrix}$.

If possible, diagonalize M, i.e., write M = PDQ, where D is diagonal.

P =________________________ **D =** ______________________ **Q =** _____________________

Problem 6: Given the eigenvalues find the eigenvectors

Given that **1 and 3 are the eigenvalues** of the matrix $C = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$, find the **eigenvectors** of this matrix.

If possible, diagonalize C, i.e., write C = PDQ, where D is diagonal. You **DO NOT** need to compute the inverse of a matrix. If a matrix is the inverse of a known matrix, just write it as the inverse.

P =________________________ **D =** ______________________ **Q =** _____________________

Problem 7: Compute orthogonal projections

Let $h = \begin{bmatrix} 6 \\ 12 \\ 3 \end{bmatrix}, u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(a) Compute m = **the projection of h on span(u).** (The formula should be computed numerically, but you need not simplify fractions, etc., in your answer.)

(b) Compute g = **the projection of h on span(u, v).** (The formula should be computed numerically, but you need not simplify fractions, etc., in your answer.)

(c) In general, if X and Y are orthogonal vectors with |X| = 5 and |Y| = 12, compute, if possible with this information, |X-Y|.

|X-Y|= _____________

Problem 8: Matrix of rotation by 120 degrees

(a) If T is the linear transformation of R^2 that rotates the plane by 120 degrees. What is the matrix A of this transformation?

Hint: cos 120 degrees = -1/2; sin 120 degrees = $\frac{\sqrt{3}}{2}$.

(b) What is the matrix B of the inverse of T?

(c) Is the matrix A an orthogonal matrix? **Yes? No?**

Show why.

(d) Is the matrix 2A an orthogonal matrix? **Yes? No?**

Show why.

Problem 9: Least squares solution

(a) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$ and let $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Then find the least squares "solution" of Ax = y.

Least squares solution = ________________

(b) If u is the least squares solution of Ax = y, how is the vector Au related to y and A? Tell what this relation is supposed to be and check that it is true in this case.

Please leave this space for the grader.

Problem	**Points Possible**	**Score**
1	25	
2	50	
3	20	
4	10	
5	20	
6	20	
7	20	
8	15	
9	20	
Total	**200**	