Problem 8: Determinant

Compute the determinant of M = $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 \\ 3 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}$.

Problem 9: Diagonal Matrix

B is a 3 x 3 matrix with eigenvalues 1, 5, 2 with corresponding eigenvectors X1, X2, X3, where $X1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $X2 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$, $X3 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$.

Let S be the matrix with columns X1, X2, X3. Then B can be written as a product involving a diagonal matrix D, the matrix S and the inverse of S.

a) Write down (explicitly, with numbers) the matrix D

D =

____

b) Write the product of D, S, S^{-1} that equals B.

B =

._____________________

Problem 10: Shorter Answers and True/False

a) Let A be any matrix. If X is a vector in range of A and Y is a vector in null space of A^T, then X and Y must be orthogonal. True ____ False _____

b) Is the set $\left\{\begin{bmatrix}1\\2\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$ linearly independent? Yes ________ No ______

c) If A, B, C are nonsingular nxn matrices, and the matrix M = ABC, then

$B = A^{-1} MC^{-1}$.

True _______ False _______ Reason:

d) Every singular nxn matrix has 0 as an eigenvalue.

True _______ False _______ Reason:

e) For any two n x n matrices, det (AB) = det (BA).

True _______ False _______ Reason:

f) If a matrix N is an orthogonal matrix, then $N^{-1} = N^T$..
True _______ False _______ Reason:

g) If A is a 2x2 matrix with eigenvalues 1 and 10, then the matrix 3A has eigenvalues 9 and 90.

True _______ False _______ Reason:

h) The matrix $P = \begin{bmatrix}1 & 2 & 2 & 3\\0 & 5 & -1 & 2\\0 & 0 & 2 & 0\\0 & 0 & 0 & 7\end{bmatrix}$ is similar to the matrix $\begin{bmatrix}5 & 0 & 0 & 0\\0 & 7 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

True _______ False _______ Reason:

Problem 11: Shorter Answers and True/False

(a) Let A be a 3 x 15 matrix.

- What are the possible values of the rank of A?

____

- What are the possible values of the nullity of A?

____

(b) True or False. If B is a matrix with nullity = 0, the columns of B are linearly independent. True or False ___________

- Why?

(c) Let $v_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$. Suppose that X is a vector in R^3 and that we know the dot products of X with these vectors, namely:

$$X^T v_1 = 2, X^T v_2 = 2, X^T v_3 = 2.$$

- What is the vector X? (This should be an answer that says X = some vector with only numbers in it, not letters. Hint: There may be some orthogonal vectors here somewhere.)