Quiz 4 -Answers
Quiz 1 Question 1

Evaluate the double integral $\int\int_D xdA$, where D is the triangular region with vertices $(-3,0),(6,0),(0,3)$.

Answer: The equations for the sides of the triangles are $y=0$, $y=x+3$, $y=-(1/2)x+3$.

So an integral is $\int_0^3\int_{y-3}^{6-2y} xdxdy = \int_0^3 x^2/2|_{y-3}^{6-2y}dy = \int_0^3(1/2)((6-2y)^2-(y-3)^2)dy =$
$= (1/2)\int_0^3(3y^2-18y+27)dy = (1/2)(y^3-9y^2+27y)|_0^3 = (1/2)(27-81+81) = 27/2$

Quiz 1 Question 2

Using polar coordinates, write an iterated integral that will compute the volume of of the region enclosed by the surface $z^2-4x^2-4y^2=20$ and the plane $z=6$.

DO NOT EVALUATE THE INTEGRAL! Just set it up completely, with limits and the integrand in polar coordinates ready to be evaluated!

Answer: The edge of this region satisfies both equations, so $z=6$ and $36-4r^2=20$, so $4r^2=16$ and $r=2$. So the integral is $\int_0^{2\pi}\int_0^2(6-\sqrt{20+4r^2})rdrd\theta$.

Quiz 2 Question 1

Set up completely this integral in CYLINDRICAL COORDINATES but *DO NOT EVALUATE*.

Set up $\int\int\int_D xzdV$, where D is the part of the solid consisting of points inside the sphere $x^2+y^2+z^2=9$ and outside the cylinder $x^2+y^2=1$ that is contained in the first octant.

(To repeat, take this solid bounded by the sphere and the cylinder but integrate only over the part of the solid in the first octant, i.e., where x,y,z are all nonnegative.)

Answer: The points outside the cylinder have $r\geq 1$ and points inside the sphere with equation $r^2+z^2=9$ satisfy $r\leq 3$.

In the first octant, $x\geq 0$, $y\geq 0$, $z\geq 0$. So the integral is

$$\int_0^{\pi/2}\int_1^3\int_0^{\sqrt{9-r^2}}(zr\cos\theta)rdzdrd\theta$$

Quiz 2 Question 2

Set up completely this integral in CARTESIAN COORDINATES (i.e., x,y,z coordinates) but *DO NOT EVALUATE* .

Set up the integral $\int\int\int_S x^2y^2dV$, where S is the solid bounded by the planes given by these equations: $z=0,\quad y=0,\quad 2x+2y+z=2,\quad -2x+2y+z=2$.

Answer: This is a tetrahedron with four triangular faces. The edge where $z=0$ and $y=0$ meet is a segment on the x-axis, with endpoints $(1,0,0)$ and $(-1,0,0)$. Any line parallel to the x-axis intersects the two faces with equations $2x+2y+z=2$ and $-2x+2y+z=2$. So x ranges from $-(1-y-z/2)$ to $1-y-z/2$.

The third vertex on the $z=0$ plane is $(0,1,0)$, where the lines $2x+2y+0=2$ and $-2x+2y+0=2$ intersect. The vertex on the plane $y=0$ is the point $(0,0,2)$, where the lines $2x+z=2$ and $-2x+z=2$ intersect.

The edge where the two planes $2x+2y+z=2$ and $-2x+2y+z=2$ meet satisfies $4y+2z=4$, or $z=2-2y$.

So this iterated integral equals the triple integral, so is one possible answer to this question.

$$\int_0^1\int_0^{2-2y}\int_{-(1-y-z/2)}^{(1-y-z/2)}x^2y^2dxdzdy$$

Quiz 3 Question 1

Let D be the set of (x, y) given by $4(x-y)^2 + (x+y)^2 \leq 4$.

Transform the integral $\int\int_D (x-y)^2 dxdy$ to an integral in u and v using the change of variables $u = 2(x-y), v = x+y$. **Hint:** Look carefully at the relationship between u and v and the definition of D.

Draw a box around this integral. Then evaluate the new integral by any method you choose.

Answer: By substituting the expressions for u and v, we see the domain D is mapped to the domain $u^2 + v^2 \leq 4$ a disk of radius 2,

Also the function $(x-y)^2$ becomes $u^2/4$.

Finally, the area $dA = dxdy$ becomes $Jdudv$, where J is the Jacobian of x, y with respect to u, v. To compute this we solve for x and y so that we can take the partial derivatives. Solving for x and y in the linear equations

$$u = 2(x-y) = 2x - 2y, \quad v = x + y$$

we get $u + 2v = 4x$ and $2v - u = 4y$, so

$$x = (1/4)(u + 2v), \quad y = (1/4)(2v - u)$$

So $\partial x/\partial u = 1/4$; $\partial x/\partial v = 1/2$; $\partial y/\partial u = -1/4$; $\partial y/\partial v = 1/2$;. Thus the Jacobian equals $(1/4)(1/2) - (1/2)(-1/4) = 1/4$.

Thus the integral is the integral

$$\int\int_{u^2+v^2\leq 4} (u^2/4)(1/4)dudv$$

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This can be evaluated using polar coordinates as:

$$(1/16)\int_0^{2\pi}\int_0^2 (r^2\cos^2\theta)rdrd\theta = (1/16)(r^4/4)|_0^2 \int_0^{2\pi} \cos^2\theta d\theta = \pi/4$$

Quiz 3 Question 2

Describe clearly the image of the rectangle given by $0 \leq s \leq 1$ and $0 \leq t \leq 1$ under the transformation $x = -t$, $y = 2s + 2t$.

Answer: Since this is a linear mapping, the image of the square must be a parallelogram. There are two ways to answer this question. One way is to compute the images of the four vertices of the square.

The points $(0,0), (1,0), (1,1), (0,1)$ map to $(0,0), (0,2), (-1,4), (-1,2)$. These are the vertices of the image.

The other method is to find the equations of the sides. The lines with equations $t = 0$ and $t = 1$ map to the lines with equations $x = 0$ and $x = -1$. The lines with equations $s = 0$ and $s = 1$ map to the lines with equations $2x + y = 0$ and $2x + y = 2$, since $s = (1/2)(2x + y)$.