Part A. Give a Proof of ONE of these two theorems, Theorem A1 or Theorem A2.

Theorem A1: Medians

Prove that the medians of any triangle ABC are concurrent.

Theorem A2: Right angles and circles

Prove, without using the inscribed angle theorem, that if AB is a diameter of circle c, then angle ACB is a right angle if and only if C is on c.

Part B. Give answers with convincing reasoning for 2 of the problems B1 - B3. Show your work.

Problem B1: Parallels at equal distance

Let m be a line and m' a line parallel to m, with distance d from m to m'. Let n be a line and n' a line parallel to n, with distance e from n to n'. If m and n are not parallel, the four lines form the sides of a parallelogram. Prove that if d = e, then the parallelogram is a rhombus.

Problem B2: Points on Rays

Given point A' on ray OA and point B' on ray OB, if |OA| = a, |OB| = b, |OA'| = 5/a, |PB'| = 5/b, then if |AB| = k, what is |A'B'|? The answer should be an expression involving numbers and (some or all of) a and b and k.

Problem B3: Special Rectangle

Let ABCD be a rectangle, with M the midpoint of AB and N the midpoint of CD. Then MNDA is also a rectangle. Let x = |AB| and y = |CD|. Then if MNDA is similar to ABCD, what is the ratio x/y?

Part C. Answer all 3 questions concerning transformations

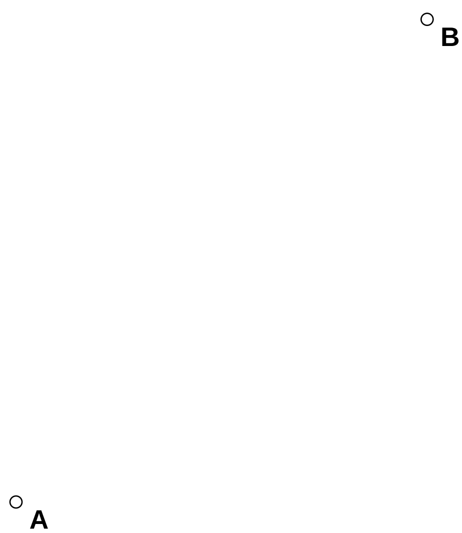
Problem C1: Isometries in the (x, y) plane.

(a) Suppose that A = (50, 10) and B = (50, 72). Let T be the translation that takes A to B and let S be the point symmetry (half-turn) with center A. Tell precisely what isometry is ST.

(b) Also, for the same S, if R is reflection in the y-axis, tell precisely what isometry is RS.

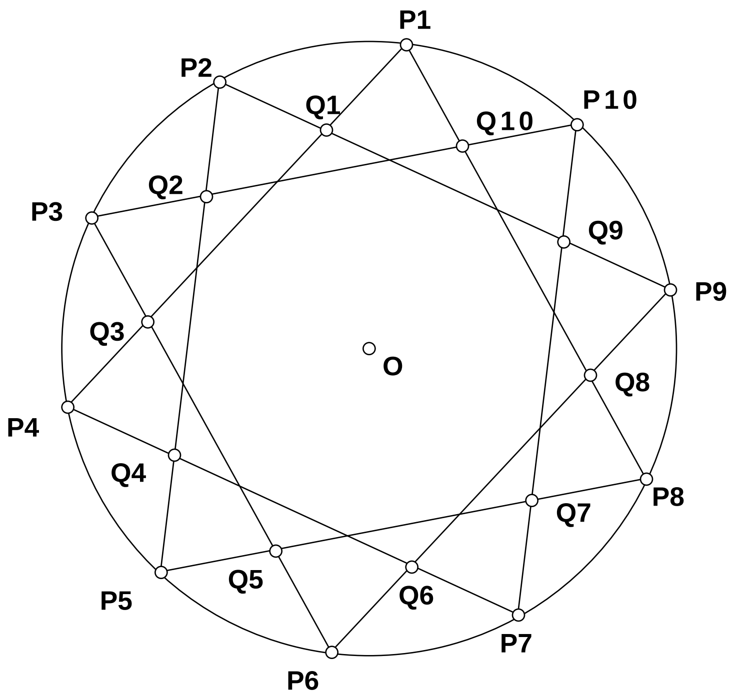
Problem C2: Construction Problem for Isometries

Given points A and B, let S be rotation by 60 degrees with center A and let T be rotation by 120 degrees with center B. Tell what is U = ST and construct (with straightedge and compass) the geometric data needed to define this transformation directly. (Geometric data means centers, mirror or invariant lines, vectors, etc.)



Problem C3: Questions about an inscribed figure

In the figure below, all the chords have equal length. Recall the notation for rotation with center Z by angle a is Za.



(a) List briefly all symmetries of this figure, telling the number of each kind.

(b) If L1 is line reflection in line OP1 and L4 is line reflection in line OP4, tell precisely what isometry is the product L1L4.

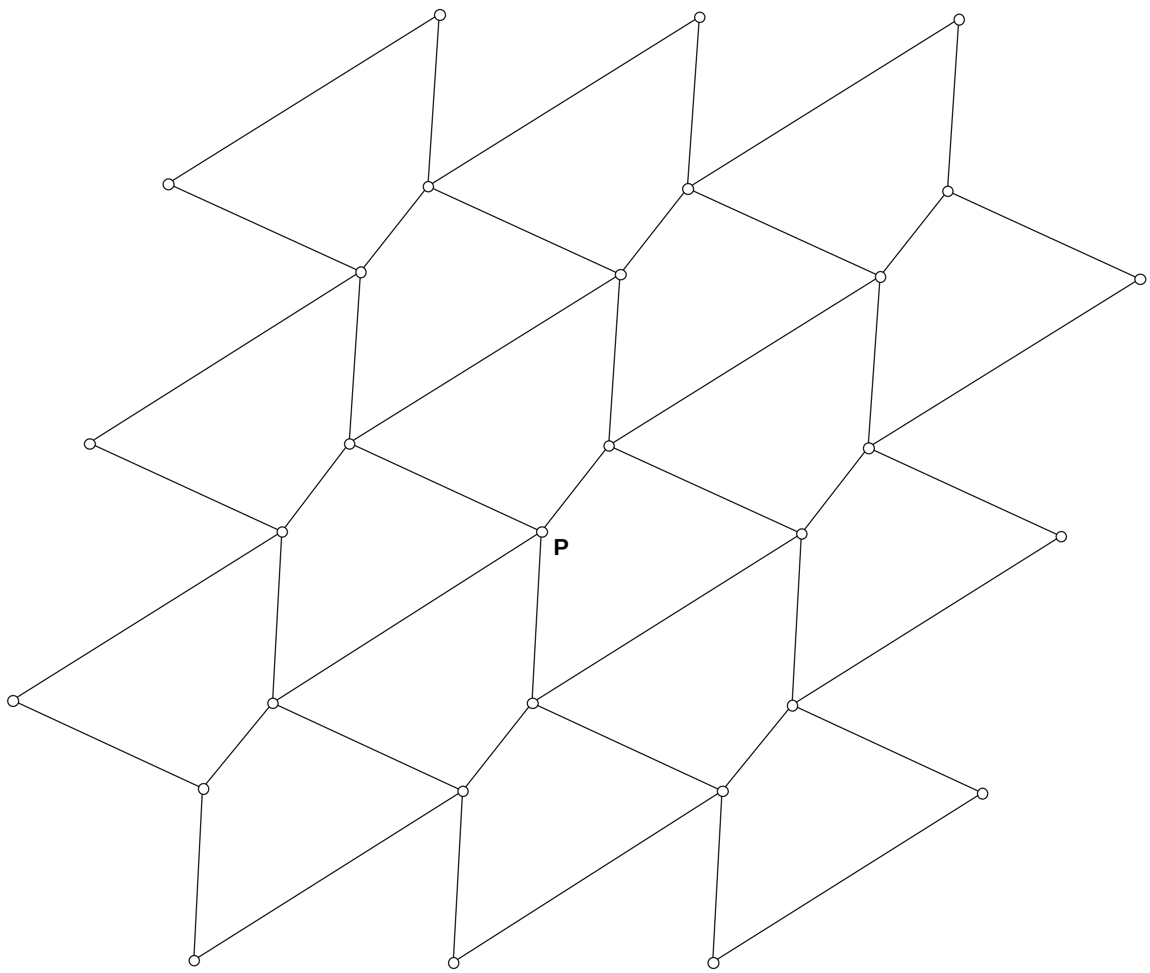
(c) Continuing with L1 and L4 and with L5 = line reflection in OP5, tell precisely what isometry is the product L1L5L4.

(d) Tell precisely what isometry is the product MN, where M = reflection in line P1P8 and N = reflection in line P1P4.

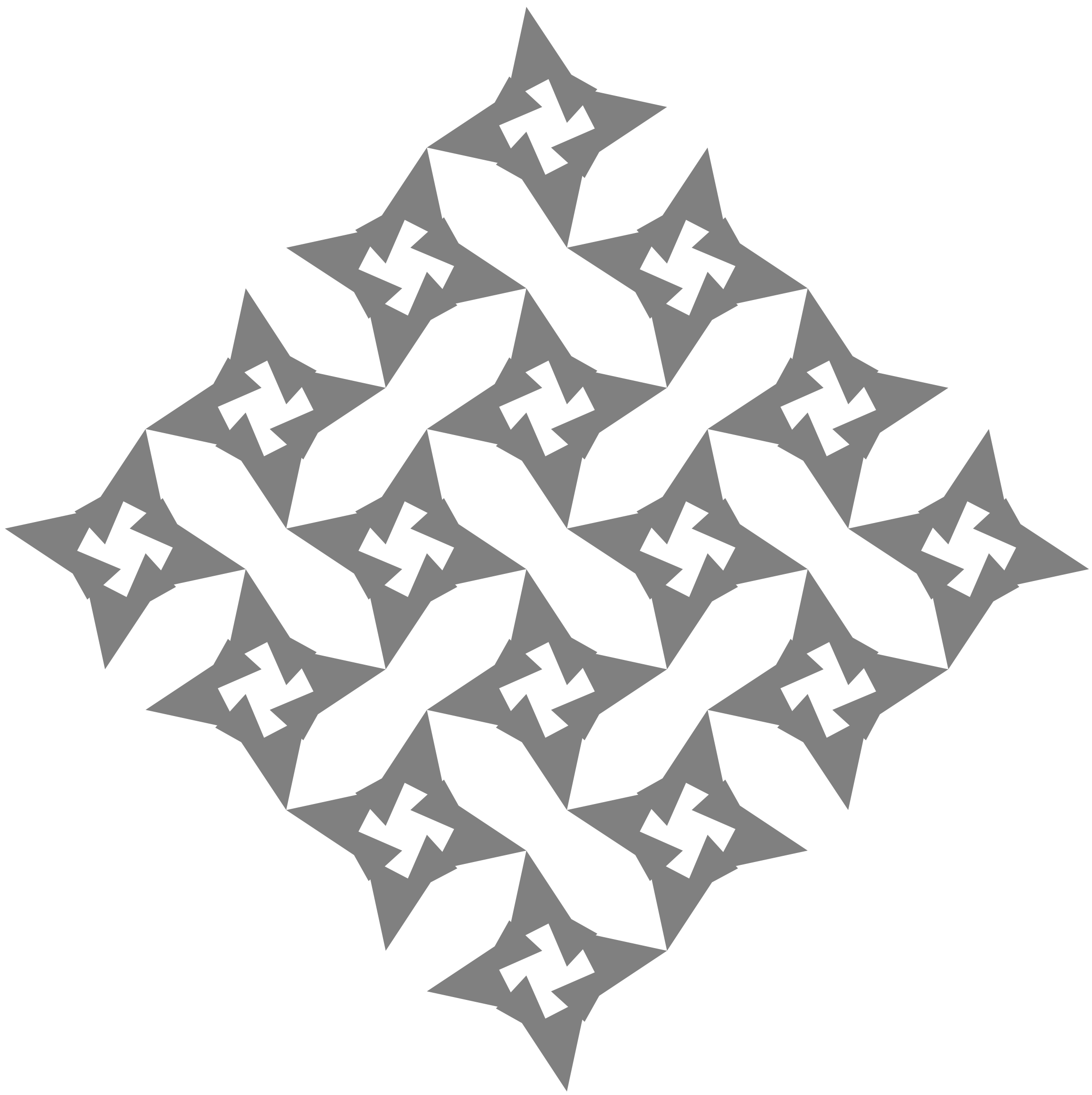
Section D. Analyze the symmetries of the patterns D1 and D2.

In each case imagine the design being continued by translations to cover the whole plane. The **specific instructions** are to mark the centers of rotation as we did on homework (squares for 90 degrees, rhombi for 180 degrees, triangles for 120 degrees, etc.), draw mirror lines as solid lines and glide reflection lines as dashed lines. Finally, pick one center and circle it and circle all other centers that are obtained from the original point by a translation that is a symmetry of the pattern. You do NOT have to CONSTRUCT the centers and lines, but draw them carefully.

Problem D1. Tessellation



Problem D2.

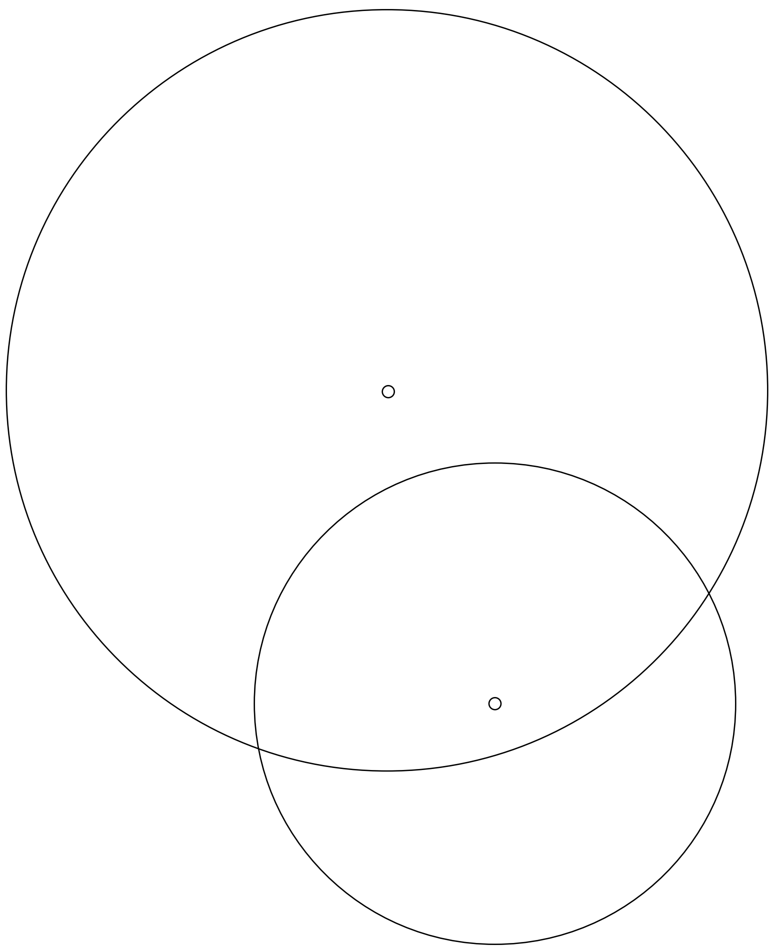


Section E. Carry out 2 of the Constructions E1 - E3.

Use Straightedge and Compass and leave your construction marks showing.

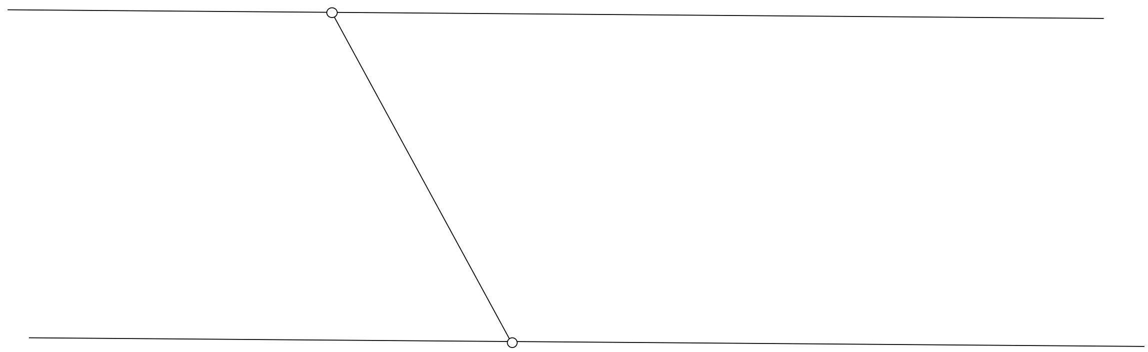
Construction E1: Common tangents

Construct the common tangents of these two circles. The centers are shown.



Construction E2: Circle tangent to 3 lines

Construct a circle tangent to all 3 lines.



Construction E3: Circle through Point

Construct a circle tangent to both lines that passes through point F.