TLP Geometry Assignment 1 (Turn in at the Beginning of Class, Monday 8/5)

What is the purpose of this assignment?

This set of problems provides a review of some basic ideas about area that will be needed next week. Most of "the facts" will be familiar, though some of you may be a bit rusty. You should emerge with

- a fresh reminder of "the facts" about area of some simple shapes
- some connections and some implications of these facts
- a visual method of explaining area relations to students and yourselves
- a foundation for class discussions about area, size, etc.

What are you asked to do? What do you turn in and when?

- **Read the Area Activities and Questions. Do all the activities and answer all the questions 1-10.** You are asked to take some measurements, answer some questions, and cut out or draw some figures. Some of the work should be written on the sheet itself. Some of the work, especially the drawings and cutouts, should be taped to or drawn on an extra Answer Sheet.
- **Answer the Summary and Reflection Questions.**
- (Optional) Answer one or more of the Challenge Questions

Reading Assignment (getting an advance look)

You will get two CMP books that we will be analyzing during the Geometry Week. You are strongly encouraged to read the introductory and orientation sections of each book before we take it up in class. However, you should *NOT* feel you need to master the book in advance. We will be working through the activities during the week. The first book that we work with will be *Stretching and Shrinking*. The second will be *Wrapping and Filling*.

YOUR GEOMETRY TOOLS

Since you will be drawing and cutting both in and out of class, you should bring with you at all times a **RULER** and **SCISSORS**. *Optionally*, you may find it useful to have colored pencils or pens, a protractor and a compass.

## Area Activities and Questions

A. Rectangle Area

1. Rectangle Area Example (Answer These Questions by Writing or Drawing on this Page)

- What is the area of each rectangle? Show how you computed the areas.
- Suggest a way to help a student understand how to the compute the areas. Add something to the drawing to show this idea.
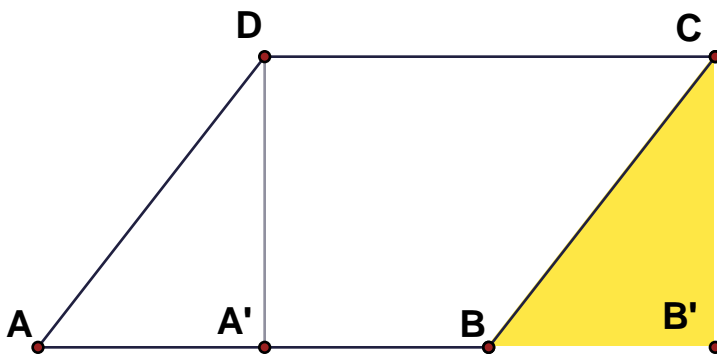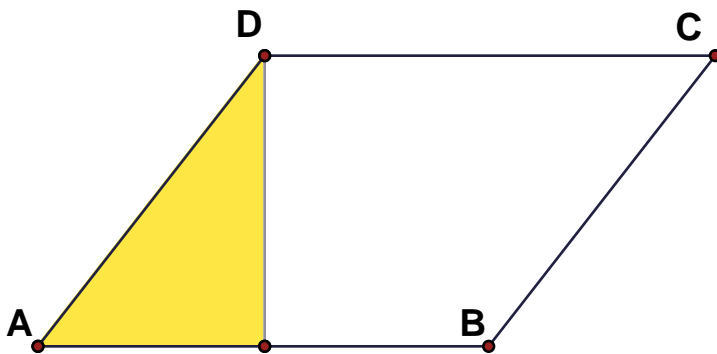- Why do you think more than one rectangle is included on this page?

7 cm

7 cm

5 cm

5 cm

7 cm

5 cm
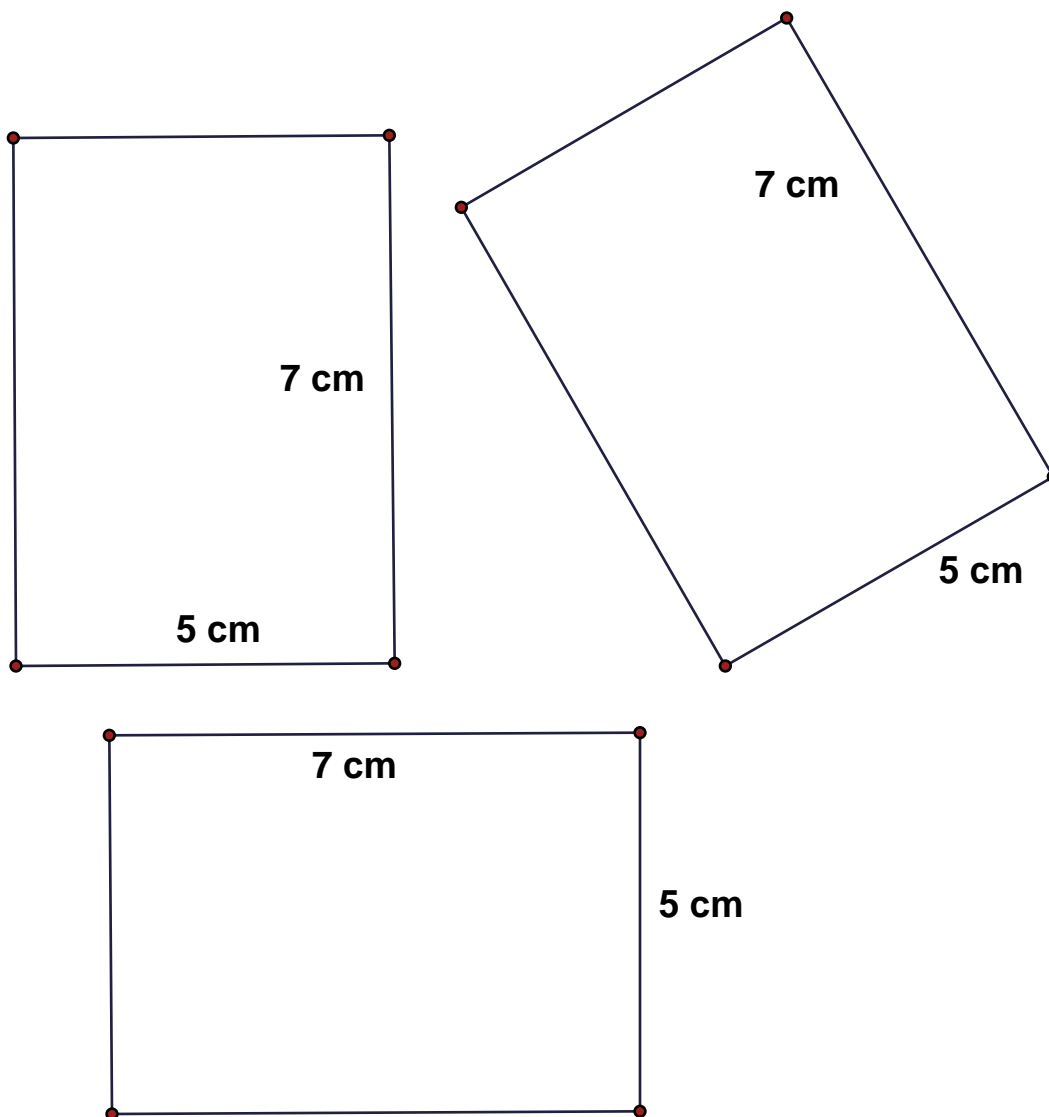
B. Parallelogram Area

The corner cut dissection (*)
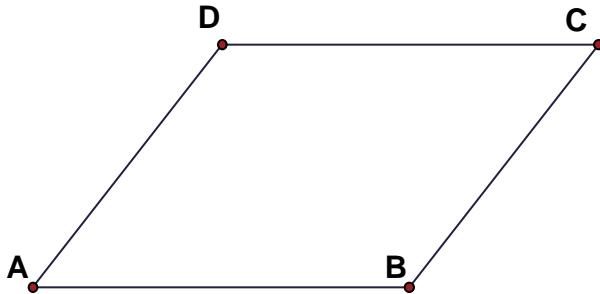
One method to deduce the area of a parallelogram is by dissection, i.e., cutting up the parallelogram and reassembling it as a rectangle. Here is a picture of how this works.

- The usual way for computing the area of a parallelogram is a formula in terms of **BASE** and **HEIGHT**. Give a definition of base and height and write an area formula using base and height.

Parallelogram Activities

2. Cut One Out! Yes, with scissors! (Tape your cutout to the Answer Sheet)

- Draw carefully on a piece of paper a parallelogram (you can use ruled paper or graph paper if that helps). Then crease along the line DA' as in the figure above. Then cut off the corner and see how it fits on the other end. Tape the corner on and bring the new rectangle to class.
- Do you think students will remember and understand this construction better if they really cut out a parallelogram?
- What method did you use to draw the parallelogram accurately? Can you think of other methods?

3. Who's on Base? (Write your measurements and computation on this page.)

The parallelogram below is the same as the ones on the previous page, but in a different position. This points out that we can just as well take BC as the base as AB.

- Measure as accurately as possible the base and height of this parallelogram using BC as the base. Compute the area from these measurements.
- Measure as accurately as possible the base and height of this parallelogram using AB as the base. Compute the area from these measurements.
- Compare your area computations (no fudging!). How accurately do you think students will be with such measurements?
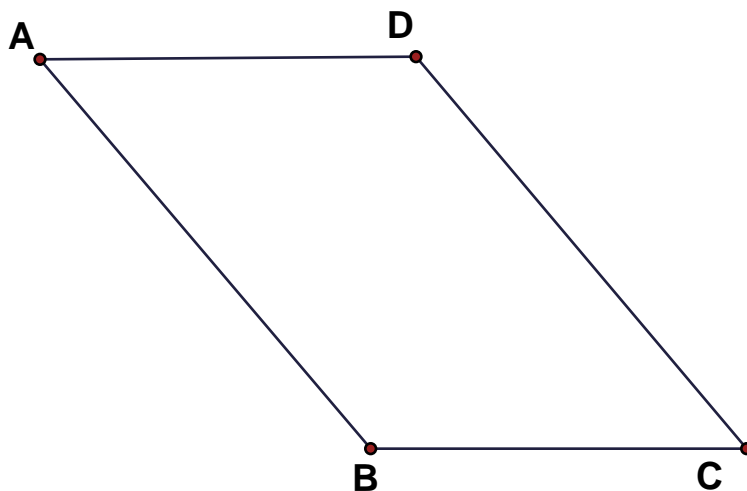
### C. Triangle Area by "doubling the triangle"

The area of a triangle can be related to the (known) area of a parallelogram by the following bit of cutting and pasting.

Start with any triangle. Make an exact second copy of the triangle.
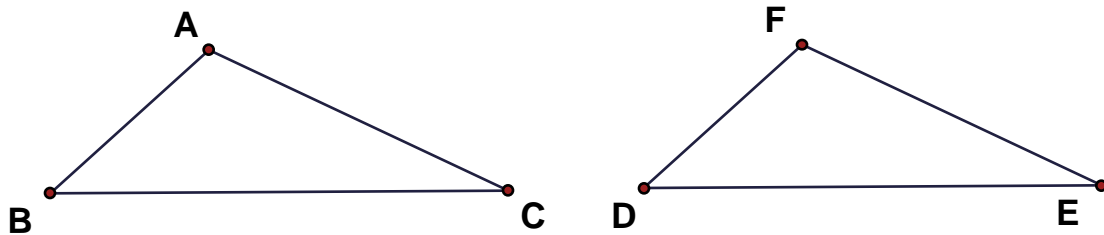
Turn one triangle over and attach to the other to form a parallelogram.

- How can you convince a friendly skeptic that this is indeed a parallelogram?
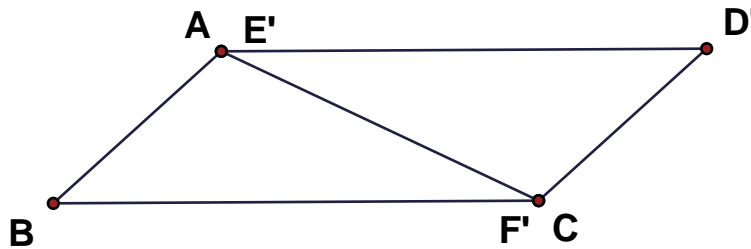
Therefore, the area of the triangle is one-half of the area of the parallelogram.

I.   How can you compute the area of a triangle if you know the base BC and the altitude (the height or altitude of the triangle is the height of this parallelogram)?

Triangle Activities and Problems

4. Cut one out and draw. (Draw this on the answer sheet.)

Cut out a triangle from cardboard. Make sure that the lengths of the 3 sides are all different for your triangle. On the Answer Sheet, use the cardboard template to draw two triangles to form a parallelogram as in the figure above. Is there more than one way to do this? Draw as many parallelograms as you can by "doubling" your triangle.

5. Shearing Triangles (Write Answer on This Sheet)

Starting with a triangle ABC, think of AB as the base. Then move C along a line parallel to AB. This produces triangles such as ABC, ABD, ABE in the figure.
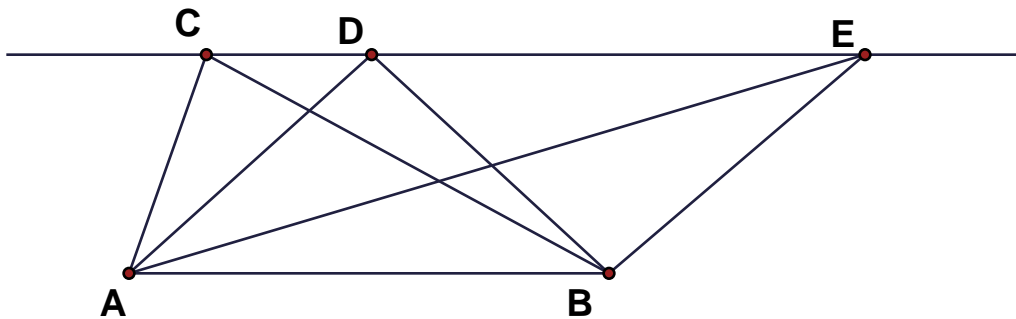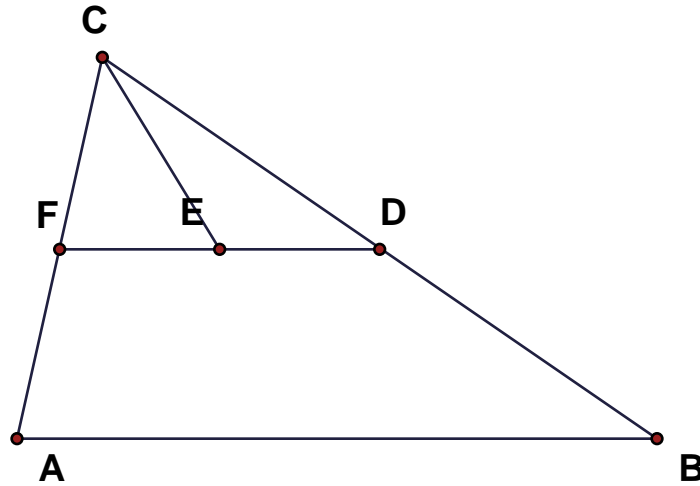

- Explain why each of these 3 triangles has the same area.

6. An Alternative Area Dissection to Cut Out (Tape your work to Answer Sheet)

Cut out any triangle ABC. Mark the midpoints D of side BC and F of side CA and then midpoint E of DF. Make the cuts DF and CE.

- Now reassemble the 3 pieces as a parallelogram with base AB.
- How do you know your figure is a parallelogram?

7. The area formula (Write your measurements and computation in the table on this page)

Measure the triangle below and compute the area 3 ways. First compute the area using AB as the base, then again using BC as the base, finally use CA as the base. Fill in the table and observe the accuracy of your measurements and computations. Be sure to include the UNITS of measurement.

Base	Altitude	Computed Area
AB =		
BC =		
CA =		

C

A   B
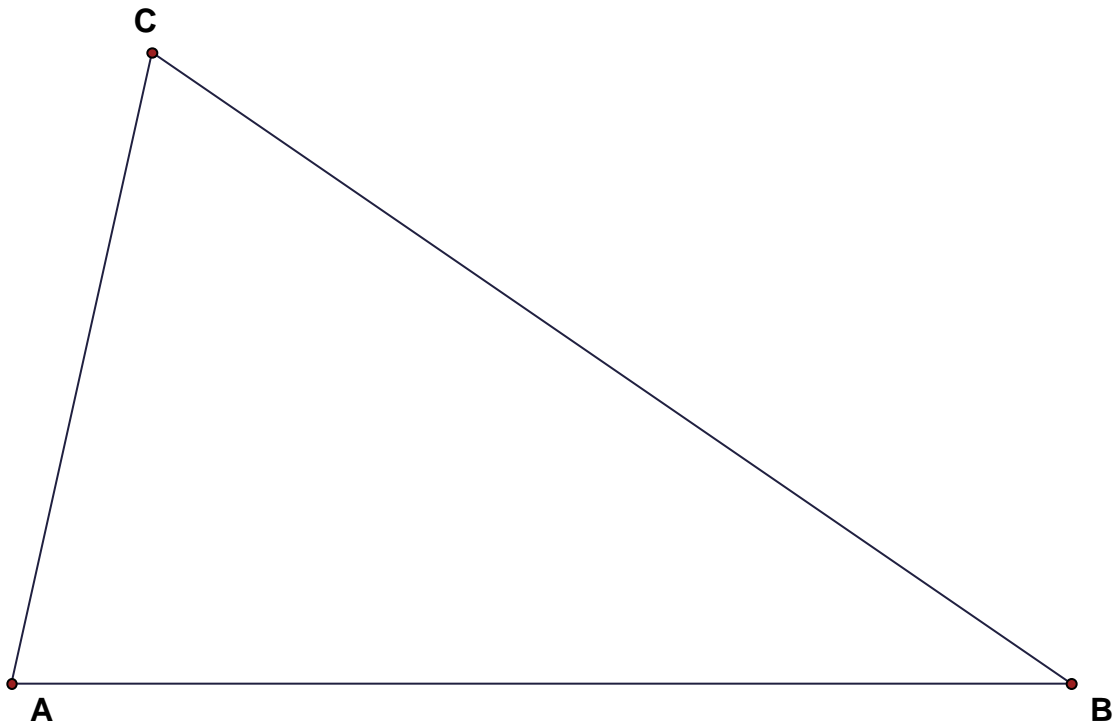
8. Comparison of Areas (Write your answers on this page)

Start with any triangle ABC and arrange 4 exact copies of the triangle in the following figure to make a bigger triangle. Suppose that triangle ABC has area = T. Figure out the areas of the polygons listed below. **(The answers should be in terms of T, such as 3T or T/2, etc.)**
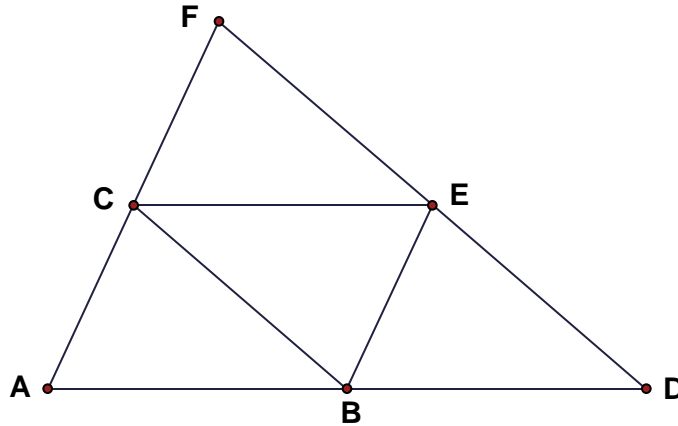
- What is the area of triangle BEC?
- What is the area of triangle ADF?
- What is the area of the quadrilateral BDEC?
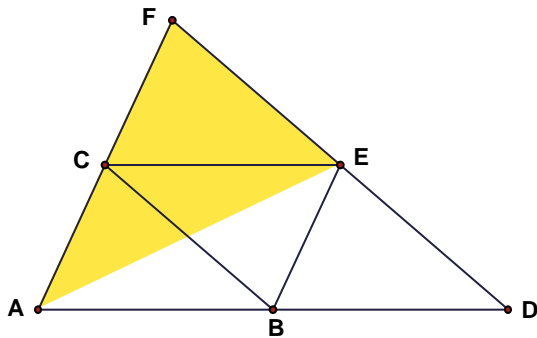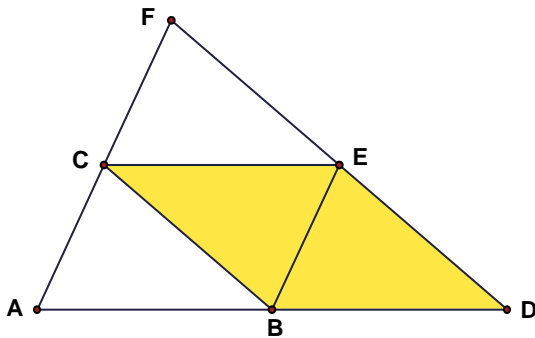
- What is the area of the triangle AEF?
- What is the area of triangle ADG? (G is the point on CE with CG/CE = 1/4.)
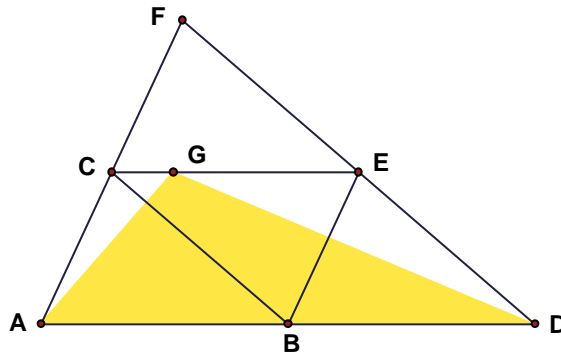
9. Areas in a Rectangle (Write your answers on this sheet)

Start with any rectangle ABCD. Draw the diagonals AC and BD.
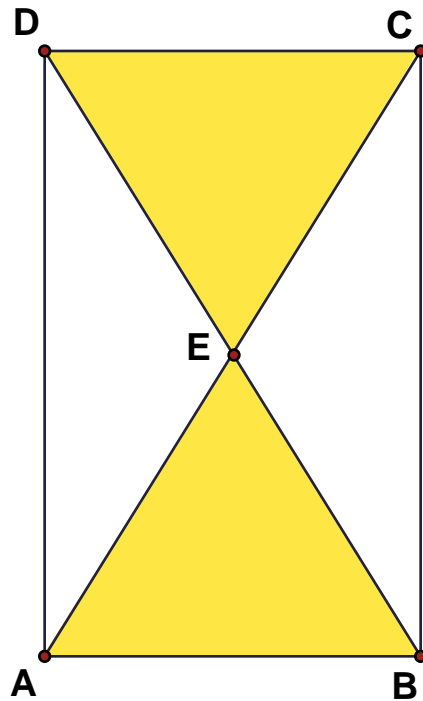- If point E is the intersection of the diagonals, which of the triangles ABE, BCE, CDE, DAE have the largest area?
- Which triangles have the largest perimeter?
- How does this change if BC is less than AB?
- How can you convince the friendly skeptic that your answer is correct?

10. Areas of Trapezoids (Cut out or draw on the Answer Sheet)

Find the definition of a trapezoid online, in a dictionary or a math book. Use some of the ideas from this handout to find a visual method to compute the area of a trapezoid (in the same spirit as the methods above for the parallelogram and the triangle). Even better, find more than one way to compute the area.

Summary and Reflection Questions (Answer on this sheet)

11. Parallelogram Area

• Tell how to compute the area of a parallelogram.

• What is meant by the height of a parallelogram?

• If you gave students a drawing of a parallelogram and asked them to measure the height, would it be possible for students to produce more than one correct answer? How many correct answers would be possible? Explain.


12. Triangle Area

• Tell how to compute the area of a triangle.

• What meant by the height (or altitude) of a triangle?

• If you gave students a drawing of a triangle and asked them to measure the altitude, would it be possible for students to produce more than one correct answer? How many correct answers would be possible? Explain.

13. Deep Questions about Cutting Corners

This question uses the figure at the start of Section B. The corner cutting argument for area of parallelogram ABCD is pretty slick and convincing, but a skeptic could ask some questions before believing that A'B'CD is a rectangle.

[If you feel no reasoning is necessary, try the same corner cutting argument on a sphere!]

•   What do you have to know about the lengths BC and AD to see that the edge of the triangle piece matches exactly at the other end?

•   What do you know about the angles of ABCD to know that the 3 points A, 'B, B' lie on a straight line?

14. Can One Always Cut Corners?

One important consequence of the corner cutting and base and height formula for area is that if two parallelograms have the same base and the same height, they have the same area. But is there a hole in our reasoning? Look at this example.
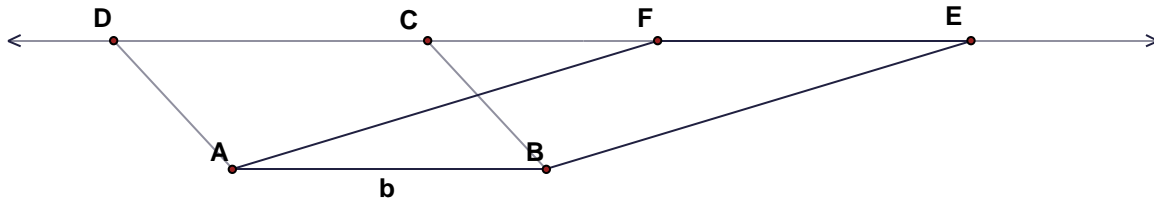
In this case, the parallelograms ABCD and ABEF have the same base AB. Also, since C, D, E, F are all on a line parallel to AB, the heights are the same, so they have the same area. But wait! Does base times height equal area in this case? Try the corner cutting method on parallelogram ABEF below.
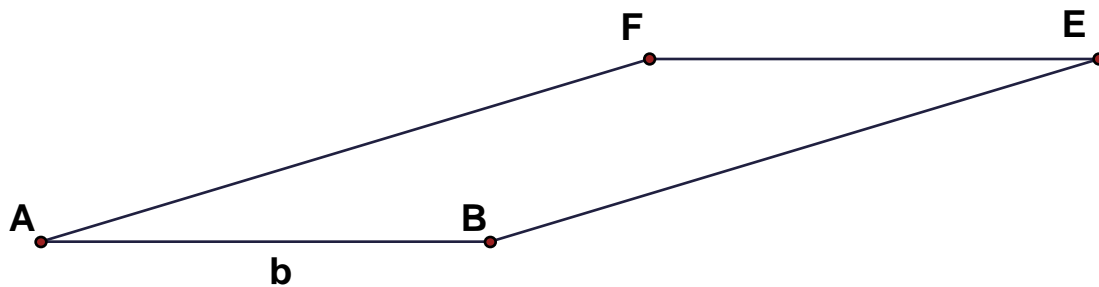
- What goes wrong? Is the formula still true?

- If so, what reason can you give?

- For a middle school teacher, what are the teaching implications of this situation?

(Optional Challenge) Can you cut apart this parallelogram and reassemble it as a rectangle with the same base and height?

Challenge Problems

I. Two Squares into One (Optional Challenge*)

We will produce an answer to this problem in class Monday.

- Draw two squares of the same size. Cut the squares into pieces and reassemble the pieces into a single square.

- If the side length of the original squares is 1 unit, what is the side length of the new single square? Explain how you know this because of area.

- Draw or tape your solution on a page and turn it in.

II. Dividing a Square Cake into 5 Pieces (Optional Challenge***)

This problem will be left open for a while for you to think about. It is possible that you can look up an answer somewhere. Please don't spoil the fun by doing that.

Suppose you have made a lemon cake that has a square base with 8-inch sides and that is 2 inches tall. This cake has chocolate icing on top and lemon icing on the sides. Explain how you can divide this cake into 5 pieces so that each piece has the same amount of cake and the same amount of each kind of icing.