Assignment 3 (Due Wednesday at the beginning of class)

Problem 1. Reasoning about Similar Rectangles

The Problem below asks for a convincing explanation of a key fact (actually two key facts, (a) and (b)) about similar rectangles. In answering, please answer **twice**, using two approaches.

Answer using a convincing example. In the first, draw a figure and give a numerical example that would be useful in explaining what is going on (to a kid or an adult who is not excessively suspicious). A good example can serve as a rather convincing answer if the general pattern is clear in the example. Even if you are a sophisticate who sees the general answer right away, still please give a good example with specific numbers.

Answer for the general case. Then give a second general answer that would satisfy a friendly skeptic who is a lot more skeptical or who wants you to spell out the general situation. This can be either very convincing geometrical or visual reasoning, or a more algebraic argument.

Problem: In each part we have two rectangles ABCD and A'B'C'D' and write the length and width of ABCD as L = |AB| and W = |BC| and the width and length of A'B'C'D as 'L = |A'B'| and W' = |B'C'|.

(a) Suppose rectangles ABCD and A'B'C'D' are similar with scaling factor K. (The vertices A', B', C', D' are the vertices corresponding to A, B, C, D.) Convince the friendly skeptic that the "internal ratios" of length/width are equal: L/W = L'/W'.

(b) Now suppose that you have two rectangles ABCD and A'B'C'D' that you don't (yet) know are similar. But you do know L/W = L'/W'. Convince the friendly skeptic that the rectangles are in fact similar. (Since they are similar, there should be a scaling factor K. What is it? How do you find K from what you know?)

Problem 2. Application to rectangular paper.

(a) Take a sheet of 8.5" x 11" paper and fold the paper in half so that the 8.5" edges are on top of each other. This new half sheet created by the fold is also a rectangle. Is this half rectangle similar to the original sheet? Explain how you can tell.

(b) In an alternate universe, the rectangular writing paper has the following property. If you fold it in half as you did in the first example, the new half sheet is similar to the original sheet. If the width of the original sheet is W, that is the length L? [Note: Such an alternate universe actually exists. For example all of Europe lives there.]