1 (10 points) Consider the vector field

$$\mathbf{F}(x, y, z) = (2xyz^2)\mathbf{i} + (x^2z^2 - 2y)\mathbf{j} + (2x^2yz)\mathbf{k}.$$

(a) Is $\mathbf{F}$ conservative? Why or why not?

(b) Let C be the path illustrated in the diagram (i.e. C is the straight line segment from $(0, 0, 0)$ to $(1, 0, 0)$, then from $(1, 0, 0)$ to $(1, 1, 0)$, then from $(1, 1, 0)$ to $(0, 1, 0)$ and finally from $(0, 1, 0)$ to $(0, 1, 1)$.) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

2 (10 points) Evaluate the double integral

$$\int\int_D e^{2x^2+9y^2}\,dA,$$

where D is the elliptical domain $2x^2+9y^2 \le 1$.

Note: if you forget how to approach this problem, then for a maximum of 7 points you may integrate:

$$\int\int_D e^{x^2+y^2}\,dA,$$

where D is the domain $x^2+y^2 \le 1$.

3 (16 points) Let $f(x, y, z) = xyz - x^2$ and $\mathbf{F}(x, y, z) = (1, xy, z^2)$. Compute each of the expressions below if the expression makes sense, and write NONSENSE! if the expression is undefined.

(a) ∇f

(b) The directional derivative of f at the point $(2, 0, 0)$ in the direction of $\mathbf{v} = (1, 1, 0)$.

(c) $\operatorname{curl} f$

(d) $\nabla \times \mathbf{F}$

(e) $\nabla \times (\nabla f)$

(f) $\nabla \cdot f$

(g) $\operatorname{grad} \mathbf{F}$

(h) $\operatorname{div} \mathbf{F}$

4 (16 points) True/False and short answer. No justification necessary.

(a) True or false? Suppose $\mathbf{F}$ is a smooth vector field and C is the unit circle traversed clockwise. If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$, then F is conservative.

(b) True or false? For every smooth function f, $\nabla \cdot (\nabla f) = 0$?

(c) True or false? If f is a smooth function and there is a unit vector $\mathbf{u}$ such that $D_{\mathbf{u}} f = 0$ at x_0, then x_0 is a critical point of f.

(d) True or false? Suppose C is a smooth positively oriented simple closed curve that bounds the region D. Then:

$$Area(D) = \int_C xy^2 dx + (x^2 y + x) dy.$$

(e) If $x = u + v$, $y = u^2 - v^2$ then the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is:

(f) Suppose x, y, z represent the side lengths of a right rectangular box, and further these side lengths change in time. Therefore the volume V of the box may be regarded a function of time. Compute an expression for $\frac{dV}{dt}$.

5 (10 points) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F}(x, y) = (x^4, xy)$ and C is the triangular curve consisting of the line segments from $(0, 0)$ to $(1, 0)$, from $(1, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(0, 0)$ traversed in the clockwise direction.

6 (18 points) Let S denote the surface defined by the equation $x^2 + y^2 + z = 4$ above the xy-plane.

(a) (5 points) Find a parametrization for S.

(b) (5 points) Is S orientable? If so, find a normal vector field along S. If not, explain.

(c) (3 points) Using your answer for (b), find the equation of the tangent plane to S at $(1, 1, 2)$.

(d) (5 points) Evaluate:

$$\int\int_S \frac{1}{\sqrt{1+4x^2+4y^2}} dS.$$

7 (10 points) Let S be the part of the sphere $x^2 + y^2 + (z-2)^2 = 8$ that lies above the xy-plane, oriented with the outward pointing normal. Use Stokes' theorem to compute

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}, \text{ where}$$

$$\mathbf{F} = (e^z x, e^z y, z e^z)$$