

1. (a) $\text{curl}\mathbf{F} = \mathbf{0}$, thus \mathbf{F} is conservative.

(b) $\mathbf{F} = \nabla f$ for $f(x, y, z) = x^2yz^2 - y^2$ Then:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(0, 1, 1) - f(0, 0, 0) = -1$$

2. Use change of coordinates, $u = \sqrt{2}x$, $v = 3y$ Then:

$$\iint_D e^{2x^2+9y^2} dA = \frac{\pi\sqrt{2}(e-1)}{6}$$

3. (a)

$$\nabla f = \langle yz - 2x, xz, xy \rangle$$

(b)

$$D_{\mathbf{v}}f(2, 0, 0) = \nabla f(2, 0, 0) \cdot \frac{\mathbf{v}}{|\mathbf{v}|} = -2\sqrt{2}$$

(c) NONSENSE!

(d)

$$\nabla \times \mathbf{F} = \langle 0, 0, y \rangle$$

(e) 0

(f) NONSENSE!

(g) NONSENSE!

(h) $x+2z$

4. (a) false

(b) false

(c) false

(d) true

(e)

$$\frac{\partial(x, y)}{\partial(u, v)} = -2u - 2v$$

(f)

$$\frac{dV}{dt} = yz \frac{dx}{dt} + xz \frac{dy}{dt} + xy \frac{dz}{dt}$$

5.

$$\int_C \mathbf{F} d\mathbf{r} = - \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA = -\frac{1}{6}$$

6. (a)

$$\mathbf{r}(r, \theta) = (r\cos\theta, r\sin\theta, 4 - r^2), \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq r \leq 2$$

(b) yes, it is orientable. $\mathbf{n} = \langle 2x, 2y, 1 \rangle$.

(c) $2(x-1)+2(y-1)+(z-2)=0$

(d)

$$\int_0^{2\pi} \int_0^2 \frac{1}{\sqrt{1+4r^2}} |\mathbf{r}_r \times \mathbf{r}_\theta| dr d\theta = 4\pi$$

7.

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r} = 0$$

where C is given by $\mathbf{r}(t) = (2\cos t, 2\sin t, 0)$, $0 \leq t \leq 2\pi$.