1 (8 points) Consider the parameterized curve in space:

$$\mathbf{r}(t) = \langle 2\cos t, 3\sin t, t\rangle, \ -\infty < t < \infty$$

Let $f(x, y, z)$ be a smooth function defined everywhere with:

$$\nabla f = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$$

Write an expression for: $\frac{d}{dt} f(\mathbf{r}(t))$ as a function of t. (This will involve the functions P, Q, and R)

2 (12 points) Consider the curve in the plane:

$$\mathbf{r}(t) = \langle t\cos t, t\sin t\rangle, \ -\pi/2 \leq t \leq \pi/2$$

This is a simple closed curve bounding a region D. Use Green's Theorem and the vector field $\mathbf{F} = -\frac{y}{2}\mathbf{i} + \frac{x}{2}\mathbf{j}$ to compute the area of D.

3 (16 points) This question is short answer. No justification will be necessary and no partial credit will be given for the items.
Let $f(x, y, z) = xyz + \sin z$, $\mathbf{F} = \langle xe^y, \sin z, x + y + z\rangle$, and $\mathbf{u}$ be the unit vector in the same direction as $\langle 3, 4, 12\rangle$.

(a) Find $\operatorname{grad} f$.

(b) Find $\nabla^2 f$.

(c) Find $D_{\mathbf{u}}f$ at (1,2,0).

(d) Find $\nabla \times \mathbf{F}$.

(e) Let C be the straight line path from $(0, 0, 0)$ to $(1, 1, \pi)$. Find $\int_C \nabla f \cdot d\mathbf{r}$.

(f) Let S be the surface of the sphere of radius 2 centered at the origin with outward pointing normal. Find $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

(g) Find $\operatorname{div} \operatorname{curl} \mathbf{F}$.

(h) Let C be the circle $\{(x, y, z) | x^2 + y^2 = 1,\ z = 0\}$, and let E be all of x, y, z space except for C. Is E simply connected?

4 (14 points) Let C be the simple closed curve parameterized as:

$$\mathbf{r}(t) = \langle 2\cos t, 2\sin t, 2\sin t \rangle,\ 0 \leq t \leq 2\pi$$

Notice C is the boundary of a surface parameterized as:

$$\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, r\sin\theta \rangle,\ 0 \leq r \leq 2,\ 0 \leq \theta \leq 2\pi$$

Let $\mathbf{F} = \langle ye^x, e^x + z^2, \sin y + x \rangle$. Use Stoke's Theorem to find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

5 (10 points) Let $E = \{(x, y, z) |\ x^2 + y^2 \le 1,\ -1 + x^2 + y^2 \le z \le 1 - x^2 - y^2\}$. Let S be the boundary surface of E with outward pointing normal. Let $\mathbf{F} = \langle xy^2, x^2y - \sin z, x^3 + z \rangle$. Use the Divergence Theorem to find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.