

1. Find the surface area of the surface $z = \frac{1}{2}(x^2 + y^2)$ over the region $D = \{(x, y) | x^2 + y^2 \leq 1, -x \leq y \leq x\}$.

2. Suppose we wish to make a thin metal washer with constant density, ρ g/cm.², with outer radius 2 cm., and inner radius a cm. for some constant a , $0 < a < 2$. We represent the washer by the region in the x, y plane, $D = \{(x, y) \mid a^2 \leq x^2 + y^2 \leq 4\}$.
- (a) Compute a formula for the mass of the washer, m , in terms of a and ρ .
 - (b) Compute a formula for the moment of inertia of the washer about the y -axis, I_y , in terms of a and ρ .
 - (c) If we want $m = 9$ g. and $I_y = 13$ g.cm.², what value must we choose for a ?

3. Use a change of variables to evaluate the double integral

$$\iint_D e^{2x^2+9y^2} dA,$$

where D is the elliptical domain $2x^2 + 9y^2 \leq 1$.

Note: if you forget how to approach this problem, then for a maximum of 7 points you may integrate:

$$\iint_D e^{x^2+y^2} dA,$$

where D is the domain $x^2 + y^2 \leq 1$.

4. Let S denote the surface defined by the equation $x^2 + y^2 + z = 4$ above the xy -plane.

(a) Find a parametrization for S .

(b) Evaluate:

$$\iint_S \frac{1}{\sqrt{1 + 4x^2 + 4y^2}} dS.$$

5. The hyperbolic sine and cosine functions are given by:

$$\cosh t = \frac{e^t + e^{-t}}{2}, \quad \sinh t = \frac{e^t - e^{-t}}{2}$$

(Notice $\cosh^2 t - \sinh^2 t = 1$) The object of this problem is to use a change of variables, $x = r \cosh t$, $y = r \sinh t$, to compute $\iint_D x - y \, dA$, where $D = \{(x, y) \mid 0 \leq y, y < x, x^2 - y^2 \leq 1\}$.

- (a) Compute the Jacobian $\frac{\partial(x,y)}{\partial(r,t)}$
- (b) Describe the region that corresponds to D in the r, t plane.
(**Solution:** $D = \{(r, t) \mid 0 < r \leq 1, 0 \leq t < \infty\}$)
- (c) Use change of variables to compute $\iint_D x - y \, dA$.