

1 (14 points) Consider the integral:

$$\int_0^1 \int_0^{1-x} \frac{1}{\sqrt{xy}} dy dx$$

- (a) Sketch the region in the x, y plane corresponding to this integral.
- (b) Use a change of variables, $u = \sqrt{x}$, $v = \sqrt{y}$, to compute the integral.

2 (14 points) For constants, $0 < R_2 < R_1$, consider the surface, T , described by the following parameterization:

$$\mathbf{r}(u, v) = \langle (R_1 + R_2 \cos v) \cos u, (R_1 + R_2 \cos v) \sin u, R_2 \sin v \rangle$$

$$0 \leq u \leq 2\pi, \quad 0 \leq v \leq 2\pi$$

This surface resembles the surface of a doughnut; it is an example of a torus.

Find the surface area of T ; this will depend on R_1 and R_2 .

(Remember: surface area = $\iint_T 1 \, dS$)

- 3 (14 points) A wire of variable density is circular, described by the curve $x^2 + y^2 = 1$. Suppose the density is given by $\rho(x, y) = 2 + x$. Find the center of mass of the wire.
(note: you can parameterize the curve as $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, $0 \leq t \leq 2\pi$)

4 (8 points) Consider the surface: $z = xy, x^2 + y^2 \leq 9$
Call this surface S .

- (a) Find a parameterization of S in terms of parameters u and v .
- (b) Find two non parallel vectors tangent to S at the point $P = (2, 1, 2)$.
- (c) Find a parameterization of the plane tangent to S at P in terms of parameters s and t .