Families of varieties of general type: the Shafarevich conjecture and related problems

Sándor Kovács

University of Washington

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations

"To me, algebraic geometry is algebra with a kick"

-Solomon Lefschetz

ション 小田 マイビット 日 うくつ

Outline







Shafarevich's Conjecture

4 The Proof of Shafarevich's Conjecture

Generalizations

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

Curves

Riemann Surfaces

- Genus
- Topology, Arithmetic and Differential Geometry

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●



Smooth Complex Projective Curve

Compact Riemann Surface

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

・ロ・・中・・中・・日・・日・



Smooth Complex Projective Curve

Compact Riemann Surface

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations



Curves

- Riemann Surfaces
- Genus
- Topology, Arithmetic and Differential Geometry

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●



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Shafarevich Conjecture

Sándor Kovács



genus 0

genus 1

genus 2,

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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genus 1

genus 0

genus 2, ...

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

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 Shafarevich Conjecture

Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations



Curves

- Riemann Surfaces
- Genus
- Topology, Arithmetic and Differential Geometry

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• $\boldsymbol{C}\simeq\mathbb{P}$

- C is a plane cubic
- C has genus \geq 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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• Smooth curves C come in three types:

• $\mathbf{C}\simeq\mathbb{P}^1$

- C is a plane cubic
- C has genus \geq 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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- Smooth curves C come in three types:
- $\mathbf{C} \simeq \mathbb{P}^1$ (rational)
- C is a plane cubic (elliptic)
- C has genus ≥ 2 (general type)

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

- ...their fundamental groups come in three types:
- $\mathbf{C}\simeq \mathbb{P}^1$
- C is a plane cubic
- C has genus ≥ 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

- ...their fundamental groups come in three types:
- $\mathbf{C} \simeq \mathbb{P}^1$
- C is a plane cubic
- C has genus ≥ 2

 π_1 is trivial

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

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- $\mathbf{C} \simeq \mathbb{P}^1$
- C is a plane cubic
- C has genus \geq 2

 π_1 is trivial

 π_1 is abelian

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

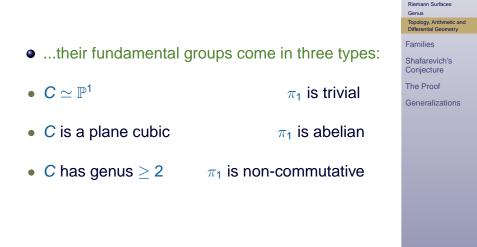
Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof



◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

Shafarevich Conjecture

Sándor Kovács

Curves

- Smooth curves C come in three types, and...
- $\mathbf{C}\simeq\mathbb{P}^1$
- C is a plane cubic
- C has genus ≥ 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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- ...rational points on them come in three types:
- $\mathbf{C}\simeq\mathbb{P}^1$
- C is a plane cubic
- C has genus ≥ 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

きょうかい 聞 ふかやえがく 西マネーロマ

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- C has genus ≥ 2

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

- ...rational points on them come in three types:
- $\mathbf{C} \simeq \mathbb{P}^1$ non-finitely generated
- C is a plane cubic
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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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- $\mathbf{C} \simeq \mathbb{P}^1$ non-finitely generated
- C is a plane cubic
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finitely generated

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

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- C has genus ≥ 2

finitely generated

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finite

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

- Smooth curves C come in three types, and...
- $\mathbf{C} \simeq \mathbb{P}^1$
- C is a plane cubic
- C has genus ≥ 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

...their curvatures come in three types:

• $\mathbf{C}\simeq \mathbb{P}^1$

- C is a plane cubic
- C has genus ≥ 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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Sándor Kovács

positively curved

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Shafarevich

Coniecture

Families

Shafarevich's Conjecture

The Proof

Generalizations

...their curvatures come in three types:

• $\mathbf{C}\simeq \mathbb{P}^1$

- C is a plane cubic
- C has genus \geq 2

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

• ...their curvatures come in three types:

- $\mathbf{C}\simeq \mathbb{P}^1$
- C is a plane cubic
- C has genus \geq 2

positively curved

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flat

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

- ...their curvatures come in three types:
- $C \simeq \mathbb{P}^1$ positively curved
- C is a plane cubic
- C has genus \geq 2

flat

negatively curved

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...their curvatures come in three types:

• $\mathbf{C}\simeq \mathbb{P}^1$

- C is a plane cubic
- C has genus \geq 2

parabolic

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Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

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- C is a plane cubic
- C has genus \geq 2

parabolic

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

elliptic

Differential Geometry of Curves

Shafarevich Conjecture

Sándor Kovács

Curves

Riemann Surfaces

Genus

Topology, Arithmetic and Differential Geometry

Families

Shafarevich's Conjecture

The Proof

Generalizations

• ...their curvatures come in three types:

- $\mathbf{C} \simeq \mathbb{P}^1$
- C is a plane cubic
- C has genus \geq 2

parabolic

elliptic

hyperbolic

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Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

Generalizations

2 Families

• Examples and Definitions

Isotrivial and Non-isotrivial Families

$y^2 = x^5 - 5\lambda x + 4\lambda$ defines a smooth family of curves parametrized by $\lambda \neq 0, 1$.

 $X = (y^2 = x^5 - 5\lambda x + 4\lambda) \subseteq \Lambda^3_{x,y,\lambda}$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

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ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

 $y^2 = x^5 - 5\lambda x + 4\lambda$ defines a smooth family of curves parametrized by $\lambda \neq 0, 1$.

$$X = (y^2 = x^5 - 5\lambda x + 4\lambda) \subseteq \mathbb{A}^3_{x,y,\lambda}$$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

ション 小田 マイビット 日 うくつ

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

ション 小田 マイビット 日 うくつ

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$$egin{aligned} X = (y^2 = x^5 - 5\lambda x + 4\lambda) \subseteq \mathbb{A}^3_{x,y,\lambda} \ &iggle \pi \ &iggle \pi \ &\mathbb{A}^1_\lambda \end{aligned}$$

ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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X π \mathbb{A}^1

ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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 $\mathbb{A}^2_{\mathbf{x},\mathbf{y}} \supseteq \mathbf{X}_{\lambda} = \pi^{-1}(\lambda) \subseteq \mathbf{X}$ π $\lambda \in \mathbb{A}^1$

ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

B – smooth projective curve of genus *g*,
 △ ⊆ *B* – finite set of points.

- A family over B \ ∆, f : X → B \ ∆, is a surjective (flat) map with equidimensional, connected fibers.
- A family is smooth if $X_b = f^{-1}(b)$ is smooth for $\forall b \in B$.

Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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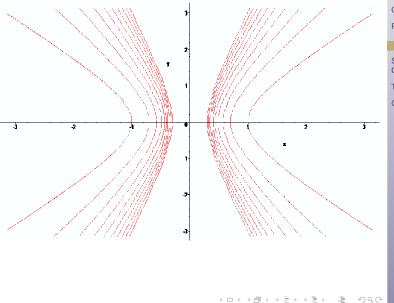
Generalizations

2 Families

• Examples and Definitions

Isotrivial and Non-isotrivial Families

Isotrivial Family



Shafarevich Conjecture

Sándor Kovács

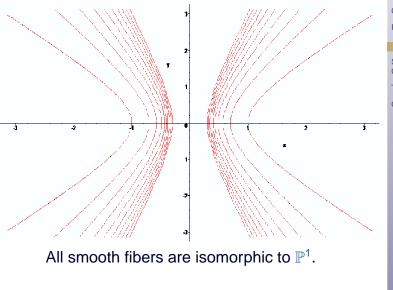
Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

Isotrivial Family



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Shafarevich Conjecture

Sándor Kovács

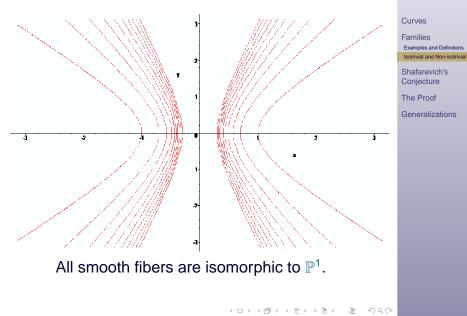
Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

Isotrivial Family



Shafarevich Conjecture

Sándor Kovács

• The family defined by $y^2 = x^5 - 5\lambda x + 4\lambda$ is not isotrivial, because

$$X_{\lambda_1}
ot\simeq X_{\lambda_2}$$

for $\lambda_1 \neq \lambda_2$.

A family f : X → B is isotrivial if there exists a finite set Δ ⊂ B such that X_b ≃ X_{b'} for all b, b' ∈ B \ Δ.

Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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Warm-Up Question: Let $q \in \mathbb{Z}$. How many isotrivial families of curves of genus q over B are there?

Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families Examples and Definitions Isotrivial and Non-isotrivial

Shafarevich's Conjecture

The Proof

Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Generalizations

Shafarevich's Conjecture Statement and Definitions

Results and Connections

• Fix B, Δ as before, $q \in \mathbb{Z}, q \ge 2$. Then

 (I) there exists only finitely many non-isotrivial smooth families of curves of genus q over B \

(II) If

$2g(\textit{B})-2+\#\Delta\leq0,$

then there aren't any admissible families.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements Results and Connections

The Proof

Generalizations

・・

- Fix B, Δ as before, $q \in \mathbb{Z}$, $q \ge 2$. Then
- there exists only finitely many non-isotrivial smooth families of curves of genus *q* over *B* \ △.

Shafarevich

Conjecture Sándor Kovács

Curves Families

Shafarevich's Conjecture

Statements Results and Connections

The Proof Generalizations

(II) If

 $2g(B)-2+\#\Delta\leq 0,$

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then there aren't any admissible families.

- Fix B, Δ as before, $q \in \mathbb{Z}, q \ge 2$. Then
- (I) there exists only finitely many non-isotrivial smooth families of curves of genus q over $B \setminus \Delta$. These will be called "admissible families".

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements Results and Connections

The Proof

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Shafarevich Conjecture Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Results and Connections

The Proof

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• Note: $2g(B) - 2 + \#\Delta \le 0 \Leftrightarrow \begin{cases} g(B) = 0 & \& & \#\Delta \le 2 \\ g(B) = 1 & \& & \Delta = \emptyset \end{cases}$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Results and Connections

The Proof

Hyperbolicity

 A complex analytic space X is Brody hyperbolic if every C → X holomorphic map is constant.

- X Brody hyperbolic implies that
 - $\forall \mathbb{C}^* \rightarrow X$ holomorphic map is constant
 - ∀ T → X holomorphic map is constant, for any complex torus T = Cⁿ/Z²ⁿ.

ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements Results and Connections

Results and Connection

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ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Hyperbolicity

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ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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ション 小田 マイビット 日 うくつ

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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ション 小田 マイビット 日 うくつ

- An algebraic variety X is algebraically hyperbolic if
 - $\forall \mathbb{A}^1 \setminus \{0\} \rightarrow X$ regular map is constant

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

- A complex analytic space X is Brody hyperbolic if every C → X holomorphic map is constant.
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- An algebraic variety X is algebraically hyperbolic if
 - $\forall \mathbb{A}^1 \setminus \{0\} \to X$ regular map is constant, and
 - ∀ A → X regular map is constant, for any abelian variety (projective algebraic group) A.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Hyperbolicity: Examples

• B – smooth projective curve, $\Delta \subseteq B$ – finite subset.

> If g(B) ≥ 2, then B \ Δ is hyperbolic for arbitrary Δ.
> If g(B) = 1, then B \ Δ is hyperbolic ⇔ Δ ≠ Ø.
> If g(B) = 0, then B \ Δ is hyperbolic ⇔ #Δ ≥ 3.

• $B \setminus \Delta$ is hyperbolic $\Leftrightarrow 2g(B) - 2 + \#\Delta > 0$.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

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Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

Statements

Results and Connections

The Proof

Outline

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements

Results and Connections

The Proof

Generalizations

Shafarevich's Conjecture Statement and Definitions

Results and Connections

 The Shafarevich Conjecture was confirmed by PARSHIN (1968) for $\Delta = \emptyset$, and by

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

The Proof

- The Shafarevich Conjecture was confirmed by PARSHIN (1968) for $\Delta = \emptyset$, and by ARAKELOV (1971) in general.
- INTERMEZZO: The number field case.
 - Mordell Conjecture (1922):
 - F a number field (a finite extension field of \mathbb{Q})
 - C a smooth projective curve of genus ≥ 2 defined over F.
 - Then *C* has only finitely many *F*-rational points. This was proved by FALTINGS (1983).
 - The Shafarevich Conjecture has a number field version as well.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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- Let f : X → B be a non-isotrivial family of projective curves of genus ≥ 2. Then there are only finitely many sections of f, i.e., σ : B → X, such that f ∘ σ = id_B.
- This was proved by MANIN (1963). And again by PARSHIN using "Parshin's Covering Trick" to prove that The Shafarevich Conjecture implies The Mordell Conjecture.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

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- Shafarevich's Conjecture Statements Results and Connections The Proof

Curves Families

Generalizations

Shafarevich

Conjecture Sándor Kovács

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

The Proof

Generalizations

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture Statements Results and Connections

The Proof

Outline

4

The Proof of Shafarevich's Conjecture

- Deformation Theory
- The Arakelov-Parshin method

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

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Deformation Theory

Arakelov-Parshin method

Deformations

A deformation of an algebraic variety X is a family F : X → T such that there exists a t ∈ T that X ≃ X_t.



Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

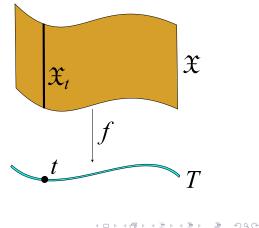
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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Deformation Theory

Arakelov-Parshin method

Deformation Type

• Two algebraic varieties X_1 and X_2 have the same deformation type if there exists a family $F : \mathfrak{X} \to T, t_1, t_2 \in T$ such that $X_1 \simeq \mathfrak{X}_{t_1}$ and $X_2 \simeq \mathfrak{X}_{t_2}$.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

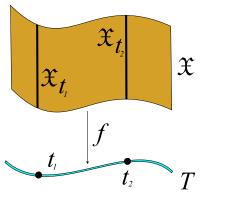
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Deformation Theory

Arakelov-Parshin method

Deformation Type

Two algebraic varieties X₁ and X₂ have the same deformation type if there exists a family F : X → T, t₁, t₂ ∈ T such that X₁ ≃ X_{t₁} and X₂ ≃ X_{t₂}.



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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Sac

Deformation Theory

Arakelov-Parshin method

Deformations of Families

• A deformation of a family $X \to B$ is a family $F : \mathfrak{X} \to B \times T$ such that there exists a $t \in T$ that $(X \to B) \simeq (\mathfrak{X}_t \to B \times \{t\})$, where $\mathfrak{X}_t = F^{-1}(B \times \{t\})$.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

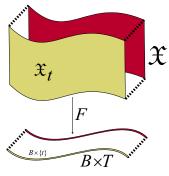
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Deformation Theory

Arakelov-Parshin method

Deformations of Families

A deformation of a family X → B is a family F : X → B × T such that there exists a t ∈ T that (X → B) ≃ (X_t → B × {t}), where X_t = F⁻¹(B × {t}).



Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Deformation Theory

Arakelov-Parshin method

Deformation Type of Families

• Two families $X_1 \rightarrow B$ and $X_2 \rightarrow B$ have the same deformation type if there exists a family $F : \mathfrak{X} \rightarrow B \times T, t_1, t_2 \in T$ such that $(X_1 \rightarrow B) \simeq (\mathfrak{X}_{t_1} \rightarrow B \times \{t\})$ and $(X_2 \rightarrow B) \simeq (\mathfrak{X}_{t_2} \rightarrow B \times \{t\})$.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

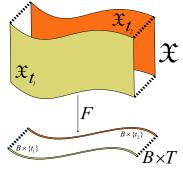
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Deformation Theory

Arakelov-Parshin method

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Deformation Theory

Arakelov-Parshin method

Generalizations

うてん 前 (声)(中)(中)(日)

Outline

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

Shafarevich

Conjecture

Generalizations

The Proof of Shafarevich's Conjecture Deformation Theory

• The Arakelov-Parshin method

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 ARAKELOV and PARSHIN reformulated the Shafarevich conjecture the following way: Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

Generalizations

- ARAKELOV and PARSHIN reformulated the Shafarevich conjecture the following way:
 - (B) *Boundedness*: There are only finitely many deformation types of admissible families.

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

Generalizations

- ARAKELOV and PARSHIN reformulated the Shafarevich conjecture the following way:
 - (B) *Boundedness*: There are only finitely many deformation types of admissible families.
 - (R) Rigidity: Admissible families do not admit non-trivial deformations.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

Generalizations

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 - (R) Rigidity: Admissible families do not admit non-trivial deformations.

• Note:

• (B) and (R) together imply (I).

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

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 - (B) *Boundedness*: There are only finitely many deformation types of admissible families.
 - (R) Rigidity: Admissible families do not admit non-trivial deformations.
 - (H) Hyperbolicity: If there exist admissible families, then $B \setminus \Delta$ is hyperbolic.

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- Note:
 - (B) and (R) together imply (I).

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

The Arakelov-Parshin method

- ARAKELOV and PARSHIN reformulated the Shafarevich conjecture the following way:
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 - (H) *Hyperbolicity*: If there exist admissible families, then $B \setminus \Delta$ is hyperbolic.
- Note:
 - (B) and (R) together imply (I).
 - (H) = (II*) and hence is equivalent to (II).

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof Deformation Theory Arakelov-Parshin method

Generalizations

Outline

5

Generalizations

• Higher Dimensional Fibers

- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results
- Higher Dimensional Bases
- Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers

Kodaira Dimensional Poets Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

• First order of business:

- Generalize the setup.
- Find a replacement for "genus" and "q ≥ 2".
 - Hilbert polynomial, h
 - Kodaira dimension, $\kappa = -\infty, 0, 1, \dots, dim$.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Shafarevich

Conjecture Sándor Kovács

Curves Families Shafarevich's Conjecture

The Proof Generalizations

Higher Dimensional Fibers Kodaira Dimension

Higher Dimensional Bases Rigidity

Admissible Families Weak Boundedness Recent Results

Where Next?

ション 小田 マイビット 日 うくつ

Outline

5

Generalizations

- Higher Dimensional Fibers
- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results
- Higher Dimensional Bases
- Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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• Plane curves:

type degree

genus

Kodaira dimension

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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٩	Plane	curves:
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type	degree	genus	dimension
₽¹	deg = 1, 2	g = 0	$\kappa = -\infty$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Kadaira

• Plane curves:

type	degree	genus	dimension
\mathbb{P}^1	deg = 1, 2	<i>g</i> = 0	$\kappa = -\infty$
elliptic	deg = 3	<i>g</i> = 1	$\kappa = 0$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Kodaira

• Plane curves:

type	degree	genus	dimension
₽¹	deg = 1, 2	<i>g</i> = 0	$\kappa = -\infty$
elliptic	deg = 3	<i>g</i> = 1	$\kappa = 0$
general type	$\text{deg} \geq 4$	<i>g</i> ≥ 2	$\kappa = 1$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

Kadaira

• Hypersurfaces in \mathbb{P}^n :

type

degree

Kodaira dimension Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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• Hypersurfaces in \mathbb{P}^n :			Curves
type	degree	Kodaira dimension	Families Shafarevich's Conjecture The Proof
	deg < <i>n</i> + 1	$\kappa = -\infty$	Higher Dimensional Fibers Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Rocent Results Higher Dimensional Bases Rigidity Where Next?

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Hypersur	faces in \mathbb{P}^n :
----------	---------------------------

type

degree	dimension
deg < <i>n</i> + 1	$\kappa = -\infty$
deg = n + 1	$\kappa = 0$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● □ ● ● ● ●

Kodaira

• Hypersurfaces in \mathbb{P}^n :			Curves
type	degree	Kodaira dimension	Families Shafarevich's Conjecture
	deg < <i>n</i> + 1	$\kappa = -\infty$	The Proof Generalizations Higher Dimensional Fibers Kodaira Dimension
	$\deg = n + 1$	$\kappa = 0$	Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases
	deg > <i>n</i> + 1	$\kappa = \dim$	Rigidity Where Next?

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• Hypersurfaces in \mathbb{P}^n :			Curves
		Kodaira	Families Shafarevich's
type	degree	dimension	Conjecture The Proof
Fano	deg < <i>n</i> + 1	$\kappa = -\infty$	Generalizations Higher Dimensional Fibers Kodaira Dimension
	deg = n + 1	$\kappa = 0$	Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases
	deg > <i>n</i> + 1	$\kappa = \dim$	Rigidity Where Next?

• Hypersurfaces in \mathbb{P}^n :			Curves
type	degree	Kodaira dimension	Families Shafarevich's Conjecture
Fano	deg < <i>n</i> + 1	$\kappa = -\infty$	The Proof Generalizations Higher Dimensional Fibers
Calabi-Yau	$\deg = n + 1$	$\kappa = 0$	Adama Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases
	deg > <i>n</i> + 1	$\kappa = \dim$	Rigidity Where Next?

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٩	Hypersurfaces	in	\mathbb{P}^{n} :
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type	degree	dimension
Fano	deg < <i>n</i> + 1	$\kappa = -\infty$
Calabi-Yau	deg = <i>n</i> + 1	$\kappa = 0$
general type	deg > <i>n</i> + 1	$\kappa = \dim$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● □ ● ● ● ●

Kodaira

• /	Hyp	ersurfaces	in	\mathbb{P}^{n} :
-----	-----	------------	----	--------------------

type	degree	dimension
Fano	deg < <i>n</i> + 1	$\kappa = -\infty$
Calabi-Yau	deg = n + 1	$\kappa = 0$
general type	deg > <i>n</i> + 1	$\kappa = \dim$

• Products:

 $dim(X \times Y) = dim X + dim Y$ $\kappa(X \times Y) = \kappa(X) + \kappa(Y)$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

・ロト・西ト・山田・山田・山下

Kodaira

Outline

5

Generalizations

- Higher Dimensional Fibers
- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results
- Higher Dimensional Bases
- Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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• X is of general type if $\kappa(X) = \dim X$.

Recall: A curve C is of general type iff $g(C) \ge 2$

• Genus is replaced with the Hilbert polynomial.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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X is of general type if κ(X) = dim X.
 Recall: A curve C is of general type iff g(C) ≥ 2.

• Genus is replaced with the Hilbert polynomial.

Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

きょうかい 明 エルサイボット 南マイロッ

• X is of general type if $\kappa(X) = \dim X$.

Recall: A curve *C* is of general type iff $g(C) \ge 2$.

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For curves, the Hilbert polynomial is determined by the genus, so this is a natural generalization.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

Setup: Fix *B* a smooth projective curve, ∆ ⊆ *B* a finite subset, and *h* a polynomial.

• $f: X \rightarrow B$ is an admissible family if

- f is non-isotrivial
- for all *b* ∈ *B* \ ∆, *X_b* is a smooth projective variety, canonically embedded with Hilbert polynomial *h*.
- "Canonically embedded" means that it is embedded by the global sections of ω^m_{X_b}. In particular, X_b is of general type.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Outline

5

Generalizations

- Higher Dimensional Fibers
- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results
- Higher Dimensional Bases
- Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nerd?

・ロト・西ト・山田・山田・山下

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity

Where Next?

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(B) *Boundedness*: There are only finitely many deformation types of admissible families.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nerd?

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- (B) *Boundedness*: There are only finitely many deformation types of admissible families.
- (R) *Rigidity*: Admissible families do not admit non-trivial deformations.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nerd?

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- (B) *Boundedness*: There are only finitely many deformation types of admissible families.
- (R) *Rigidity*: Admissible families do not admit non-trivial deformations.
- (H) *Hyperbolicity*: If there exist admissible families, then $B \setminus \Delta$ is hyperbolic.

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Shafarevich Conjecture

- (B) *Boundedness*: There are only finitely many deformation types of admissible families.
- (R) *Rigidity*: Admissible families do not admit non-trivial deformations.
- (H) *Hyperbolicity*: If there exist admissible families, then $B \setminus \Delta$ is hyperbolic.
- (WB) Weak Boundedness: If $f : X \to B$ is an admissible family, then deg $f_* \omega^m_{X/B}$ is bounded in terms of B, Δ, h, m .

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nerd?

• Moduli Theory + (WB) \Rightarrow (B)

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Neachat Results Higher Dimensional Bases Rigidity Where Next?

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• Moduli Theory + (WB) \Rightarrow (B)

• $(WB) \Rightarrow (H)$

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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• Moduli Theory + (WB) \Rightarrow (B)

• (K____, 2002) (WB) \Rightarrow (H)

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

- Moduli Theory + (WB) \Rightarrow (B)
- (K____, 2002) (WB) \Rightarrow (H)
- Hence Weak Boundedness is the key statement towards proving (B) and (H).

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nert?

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- (K____, 2002) (WB) \Rightarrow (H)
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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Outline

5

Generalizations

- Higher Dimensional Fibers
- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results
- Higher Dimensional Bases
- Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Bacent Results

Higher Dimensional Bases Rigidity Where Next?

・ロト・西ト・山田・山田・山下

Modern History

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

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Modern History

"modern" = last 25 years

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results Higher Dimensional Bases Rigidity

Where Next?

◆□▶ ◆□▶ ◆□▶ ◆□▶ ▲□ ● ● ●

Modern History

"modern" \approx last 10 years

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

BEAUVILLE (1981):
 (H) holds for dim(X/B) = 1.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

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- BEAUVILLE (1981):
 (H) holds for dim(X/B) = 1.
- CATANESE-SCHNEIDER (1994):
 (H) can be used for giving upper bounds on the size of automorphisms groups of varieties of general type.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

- BEAUVILLE (1981):
 (H) holds for dim(X/B) = 1.
- CATANESE-SCHNEIDER (1994):
 (H) can be used for giving upper bounds on the size of automorphisms groups of varieties of general type.
- SHOKUROV (1995):

(H) can be used to prove quasi-projectivity of moduli spaces of varieties of general type.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Bacont Results

MIGLIORINI (1995): (H) holds for dim(X/B) = 2 and g(B) = 1.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

- MIGLIORINI (1995):
 (H) holds for dim(X/B) = 2 and g(B) = 1.
- K____ (1996):
 (H) holds for dim(X/B) arbitrary and g(B) = 1.



Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Where Next?

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundechess Recent Results Higher Dimensional Bases Rigidity

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- MIGLIORINI (1995):
 (H) holds for dim(X/B) = 2 and g(B) = 1.
- K____ (1996):
 (H) holds for dim(X/B) arbitrary and g(B) = 1.

Shafarevich Conjecture Sándor Kovács

Curves

Shafarevich's Conjecture

Generalizations Higher Dimensional Fibers

Kodaira Dimension Admissible Families Weak Boundedness

Recent Results Higher Dimensional Bases

Rigidity Where Next?

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The Proof

K____ (1997):
 (H) holds for dim(X/B) = 2.

- MIGLIORINI (1995):
 (H) holds for dim(X/B) = 2 and g(B) = 1.
- K____ (1996):
 (H) holds for dim(X/B) arbitrary and g(B) = 1.
- K____ (1997):
 (H) holds for dim(X/B) = 2.
- K____ (2000):
 (H) holds for dim(X/B) arbitrary.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nert?

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- MIGLIORINI (1995):
 (H) holds for dim(X/B) = 2 and g(B) = 1.
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 (H) holds for dim(X/B) arbitrary and g(B) = 1.
- K____ (1997):
 (H) holds for dim(X/B) = 2.
- K____ (2000):
 (H) holds for dim(X/B) arbitrary.
- VIEHWEG AND ZUO (2002): Brody hyperbolicity holds as well.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Neacht Results Higher Dimensional Bases Rigidity Where Nert?

• BEDULEV AND VIEHWEG (2000):

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

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• BEDULEV AND VIEHWEG (2000):

• (B) holds for $\dim(X/B) = 2$, and

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

ション 小田 マイビット 日 うくつ

• BEDULEV AND VIEHWEG (2000):

- (B) holds for $\dim(X/B) = 2$, and
- (WB) holds for $\dim(X/B)$ arbitrary.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness

Recent Results

Higher Dimensional Bases Rigidity Where Next?

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- (B) holds for $\dim(X/B) = 2$, and
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As a byproduct of their proof they also obtained that (H) holds in these cases.

ション 小田 マイビット 日 うくつ

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Bacent Results

BEDULEV AND VIEHWEG (2000):

- (B) holds for $\dim(X/B) = 2$, and
- (WB) holds for dim(X/B) arbitrary.

As a byproduct of their proof they also obtained that (H) holds in these cases.

 K____ (2002), VIEHWEG AND ZUO (2002): (WB) holds under more general assumptions.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results

BEDULEV AND VIEHWEG (2000):

- (B) holds for $\dim(X/B) = 2$, and
- (WB) holds for $\dim(X/B)$ arbitrary.

As a byproduct of their proof they also obtained that (H) holds in these cases.

- K____ (2002), VIEHWEG AND ZUO (2002): (WB) holds under more general assumptions.
- K____ (2002): (WB) implies (H).

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results

...more related results by

- FALTINGS (1983)
- ZHANG (1997)
- Oguiso and Viehweg (2001)

..and more.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results

Higher Dimensional Bases Rigidity Where Next?

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Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results

Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results

Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Bacent Results

Higher Dimensional Bases Rigidity Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Bacent Results

Higher Dimensional Bases Rigidity Where Next?

Outline

5

Generalizations

- Higher Dimensional Fibers
- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results

Higher Dimensional Bases

Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nert?

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Many details change:

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity

Where Next?

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Many details change:

• Instead of Δ being a finite subset,

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidty Where Next?

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Many details change:

 Instead of △ being a finite subset, it is a 1-codimensional subvariety of B. Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Nerd?

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Many details change:

- Instead of △ being a finite subset, it is a 1-codimensional subvariety of B.
- A family being non-isotrivial is no longer a good assumption.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Many details change:

- Instead of △ being a finite subset, it is a 1-codimensional subvariety of B.
- A family being non-isotrivial is no longer a good assumption. It has to be replaced by a family having maximal variation in moduli.

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Many details change:

- Instead of △ being a finite subset, it is a 1-codimensional subvariety of B.
- A family being non-isotrivial is no longer a good assumption. It has to be replaced by a family having maximal variation in moduli.
- Viehweg's Conjecture (2001): If there exists an admissible family with maximal variation in moduli, then (B, Δ) is of log general type.

Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

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Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

 Viehweg's Conjecture (2001): If there exists an admissible family with maximal variation in moduli, then (B, Δ) is of log general type. Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

ション 小田 マイビット 日 うくつ

- Viehweg's Conjecture (2001): If there exists an admissible family with maximal variation in moduli, then (B, Δ) is of log general type.
- K_ (2003), VIEHWEG-ZUO, (2003): Viehweg's conjecture holds for $B = \mathbb{P}^n$.

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Curves

Families

Shafarevich's Conjecture

The Proof

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- Viehweg's Conjecture (2001): If there exists an admissible family with maximal variation in moduli, then (B, Δ) is of log general type.
- K_ (2003), VIEHWEG-ZUO, (2003):
 Viehweg's conjecture holds for B = Pⁿ.
- K____(1997): If $f : X \to B$ is admissible and *B* is an abelian variety, then $\Delta \neq \emptyset$.

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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But, what about Rigidity?

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Outline

5

Generalizations

- Higher Dimensional Fibers
- Kodaira Dimension
- Admissible Families
- Weak Boundedness
- Recent Results
- Higher Dimensional Bases
- Rigidity

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases **Rigidity** Where Next?

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Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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 Let Y → B be an arbitrary non-isotrivial family of curves of genus ≥ 2, and

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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- Let Y → B be an arbitrary non-isotrivial family of curves of genus ≥ 2, and
- C a smooth projective curve of genus \geq 2.

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Curves

Families

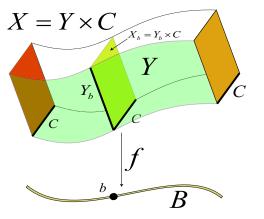
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The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Families

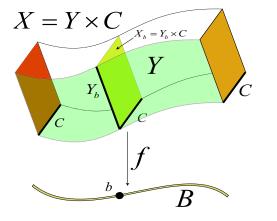
Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases **Rigidity** Where Next?

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• $f: X = Y \times C \rightarrow B$ is an admissible family, and



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Families

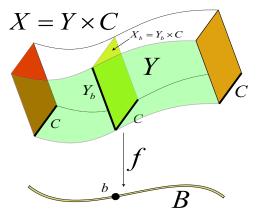
Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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- $f: X = Y \times C \rightarrow B$ is an admissible family, and
- any deformation of C gives a deformation of f.



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Curves

Families

Shafarevich's Conjecture

The Proof

• Therefore, (R) fails.

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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• Therefore, (R) fails.

Question: Under what additional conditions does (R) hold?

Shafarevich Conjecture

Sándor Kovács

Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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- Therefore, (R) fails.
- Question: Under what additional conditions does (R) hold?
- K____ (2004), VIEHWEG-ZUO, (2004): Rigidity holds for strongly non-isotrivial families.

Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodara Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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- Therefore, (R) fails.
- Question: Under what additional conditions does (R) hold?
- K____ (2004), VIEHWEG-ZUO, (2004): Rigidity holds for strongly¹non-isotrivial families.

¹unfortunately the margin is not wide enough to define this term.

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Curves

Families

Shafarevich's Conjecture

The Proof

• Work in progress:

Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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- Work in progress:
 - Geometric description of strongly non-isotrivial families.

Shafarevich Conjecture

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

- Work in progress:
 - Geometric description of strongly non-isotrivial families.
 - Joint work with Stefan Kebekus (Köln): Families over two-dimensional bases.

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations Higher Dimensional Fibers Kodaira Dimension Admissible Families Weak Boundedness Recent Results Higher Dimensional Bases Rigidity Where Next?

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Acknowledgement

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Curves

Families

Shafarevich's Conjecture

The Proof

Generalizations

This presentation was made using the beamertex LATEX macropackage of Till Tantau. http://latex-beamer.sourceforge.net

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