Corrections to
Introduction to Smooth Manifolds (Second Edition)
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(8/8/16) Page 6, just below the last displayed equation: Change $\varphi([x])$ to $\varphi_{n+1}[x]$, and in the next line, change x^i to x^{n+1}. After "(Fig. 1.4)," insert "with similar interpretations for the other charts."

(8/8/16) Page 7, Fig. 1.4: Both occurrences of x^i should be x^{n+1}.

(12/19/18) Page 9, proof of Theorem 1.15: In the second line of the proof, replace "For each j" with "For each $j \geq 0$." Then in the fourth-to-last line, replace "positive integers" by "nonnegative integers."

(1/15/21) Page 13, line 1: Delete the words "and injective."

(1/18/21) Page 20, Example 1.31: There are multiple errors in this example. Replace everything after the first two sentences by the following: For each $i = 1, \dots, n+1$, let $\left(U_i^\pm \cap \mathbb{S}^n, \varphi_i^\pm\right)$ denote the graph coordinate charts we constructed in Example 1.4. For any distinct indices i and j and any choices of $\pm$ signs, the transition maps $\varphi_i^\pm \circ \left(\varphi_j^\pm\right)^{-1}$ and $\varphi_i^\pm \circ \left(\varphi_j^\mp\right)^{-1}$ are easily computed. For example, in the case $i < j$, we get the following formula for all u in the domain of $\varphi_i^+ \circ \left(\varphi_j^+\right)^{-1}$:

$$\varphi_i^+ \circ \left(\varphi_j^+\right)^{-1}\left(u^1, \dots, u^n\right) = \left(u^1, \dots, \widehat{u^i}, \dots, \sqrt{1-|u|^2}, \dots, u^n\right)$$

(with u^i omitted and the square root replacing u^j), and similar formulas hold in the other cases. When $i = j$, the domains of φ_i^+ and φ_i^- are disjoint, so there is nothing to check. Thus, the collection of charts $\left\{\left(U_i^\pm \cap \mathbb{S}^n, \varphi_i^\pm\right)\right\}$ is a smooth atlas, and so defines a smooth structure on $\mathbb{S}^n$. We call this its standard smooth structure. //

(6/23/13) Page 23, two lines below the first displayed equation: Change "any subspace $S \subseteq V$" to "any k-dimensional subspace $S \subseteq V$."

(9/15/19) Page 24, first full paragraph, fourth line: Change "any subspace S" to "any k-dimensional subspace S."

(12/19/18) Page 26, first line: Change $U \cap \varphi^{-1}(\operatorname{Int} \mathbb{H}^n)$ to $\varphi^{-1}(\operatorname{Int} \mathbb{H}^n)$.

(12/19/18) Page 27, last paragraph, sixth line: Change $\widetilde{U} \cap \mathbb{H}^n$ to $\widetilde{U} \cap U$.

(2/22/15) Page 29, proof of Theorem 1.46, second paragraph, line 4: Change $\varphi(U \cap V)$ to $\psi(U \cap V)$.

(10/8/15) Page 30, Problem 1-6: Interpret the formula for F_s to mean $F_s(0) = 0$ when $s \leq 1$.

(1/27/18) Page 31, Fig. 1.13: Change $\{x^n = 0\}$ to $\{x^{n+1} = 0\}$.

(3/12/18) Page 31, Problem 1-11, next-to-last line: Change $\mathbb{S}^n$ to $\mathbb{S}^n \smallsetminus \{N\}$.

(4/25/17) Page 45, second paragraph: Replace the last sentence of that paragraph with the following: "If N has empty boundary, we say that a map $F\colon A \to N$ is smooth on A if it has a smooth extension in a neighborhood of each point: that is, if for every $p \in A$ there exist an open subset $W \subseteq M$ containing p and a smooth map $\widetilde{F}\colon W \to N$ whose restriction to $W \cap A$ agrees with F. When $\partial N \neq \varnothing$, we say $F\colon A \to N$ is smooth on A if for every $p \in A$ there exist an open subset $W \subseteq M$ containing p and a smooth chart (V, ψ) for N whose domain contains $F(p)$, such that $F(W \cap A) \subseteq V$ and $\psi \circ F|_{W \cap A}$ is smooth as a map into $\mathbb{R}^n$ in the sense defined above (i.e., it has a smooth extension in a neighborhood of each point)."

(7/23/14) Page 45, last displayed equation: The first $=$ sign should be $\subseteq$.

(9/15/19) Page 46, line 9: Change "on an open subset" to "on a nonempty open subset."

(6/20/18) Page 47, proof of Theorem 2.29, second paragraph: Replace the first sentence of the paragraph by "Let $h\colon \mathbb{R}^n \to \mathbb{R}$ be a smooth bump function that is positive in $B_1(0)$ and zero elsewhere."

(2/13/22) Page 49, Problem 2-10(c): Change "an isomorphism" to "a bijection."

(1/20/22) Page 54, just after the first sentence: Insert "(The integral is a smooth function of x by iterative application of Theorem C.14.)"

(11/17/12) Page 56, first displayed equation: Change $d\iota(v)_p$ to $d\iota_p(v)$.

(1/21/21) Page 56, just below the last displayed equation: Replace "the last two equalities follow" by "the last equality follows."

(6/9/19) Page 58, proof of Lemma 3.11, next-to-last line: Change $\mathbb{H}^n$ to $\operatorname{Int}\mathbb{H}^n$.

(1/26/15) Page 68, proof of Proposition 3.21: Insert the following sentence at the beginning of the proof: "Let $n = \dim M$ and $m = \dim N$." Then in the second sentence, change (3.9) to (3.10). Finally, in the displayed equation, change F^n to F^m (twice).

(11/17/12) Page 70, two lines above Corollary 3.25: Change "Proposition 3.23" to "Proposition 3.24."

(3/5/15) Page 76, Problem 3-8: Add the following remark: "(For $p \in \partial M$, we need to allow curves with domain $[0,\varepsilon)$ or $(-\varepsilon,0]$ and to interpret the derivatives as one-sided derivatives.)"

(10/23/18) Page 78, proof of Prop. 4.1, third and fourth lines: Change $m \times n$ to $n \times m$ (twice).

(11/9/16) Page 79, proof of Theorem 4.5, fourth line: Change $\widehat{F}(p)$ to $\widehat{F}(0)$.

(12/12/21) Page 82, line 4 from the bottom: Change "This is a diffeomorphism onto its image" to "This is an open map and a diffeomorphism onto its image."

(12/12/21) Page 83, proof of Theorem 4.14, line 8: Change "no open subset" to "no nonempty open subset."

(5/4/13) Page 96, Problem 4-3: This problem probably needs a better hint. First, to get a good result, you'll have to add the assumption that $\ker dF_p \not\subseteq T_p\partial M$. After choosing smooth coordinates, you can assume $M \subseteq \mathbb{H}^m$ and $N \subseteq \mathbb{R}^n$, and extend F to a smooth function $\widetilde{F}$ on an open subset of $\mathbb{R}^m$. If $\operatorname{rank} F = r$, show that there is a coordinate projection $\pi\colon \mathbb{R}^n \to \mathbb{R}^r$ such that $\pi \circ \widetilde{F}$ is a submersion, and apply the rank theorem to $\pi \circ \widetilde{F}$ to find new coordinates in which $\widetilde{F}$ has a coordinate representation of the form $\widehat{F}(x,y) = (x, R(x,y))$. Then use the rank condition to show that $R|_M$ is independent of y.

(12/22/21) Page 100, first sentence: At the end of the sentence, change "smooth embeddings" to "smooth embeddings of smooth manifolds."

(9/8/15) Page 100, proof of Proposition 5.4, next-to-last line: Change "It a homeomorphism" to "It is a homeomorphism."

(7/8/19) Page 104, line below the proof of Theorem 5.11: Change "See Theorem 5.31" to "See Problem 5-24." [Problem 5-24 is a new problem, described later in this list. Theorem 5.31 is not appropriate in this situation because it applies only to manifolds without boundary.]

(6/9/19) Page 105, line 4 from the bottom: Change F to Φ.

(11/9/16) Page 112, Fig. 5.10: Interchange the labels M and N on the figure, to be consistent with the notation in Theorem 5.29.

(5/5/22) Page 113, line 6: Change the definition of $\widetilde{\psi}$ to $\widetilde{\psi} = \pi \circ \psi|_{V_0}$. After the end of that sentence, insert the following: "To see that $\widetilde{\psi}$ is a smooth coordinate map, let $i: V \hookrightarrow M$ be the inclusion map. Note first that for each $q \in V_0$, $x^{k+1}, \dots, x^n$ are all constant on the image of i, so the image of di_q is contained in the span of $\partial/\partial x^1, \dots, \partial/\partial x^k$. Since di_q is injective and its image has trivial intersection with $\operatorname{Ker} d\widetilde{\psi}_q$, it follows that $d\widetilde{\psi}_q \circ di_q$ is injective, so for dimensional reasons it is an isomorphism. Thus $\widetilde{\psi} \circ i$ is a local diffeomorphism by the inverse function theorem. Since it is bijective from V_0 to its image, it is a diffeomorphism and hence a smooth coordinate map for V."

(9/15/19) Page 118, Fig. 5.13: Change N to v.

(9/20/22) Page 119, third line: Starting in the middle of that line, replace the rest of the proof with the following: "For each α such that $p \in U_\alpha$, we have $f_\alpha(p) = 0$ and $v(f_\alpha) = v^n > 0$ by Proposition 5.41. Thus

$$v(f) = \sum_\alpha \big(f_\alpha(p) v(\psi_\alpha) + \psi_\alpha(p) v(f_\alpha) \big).$$

For each α, the first term in parentheses is zero and the second is nonnegative, and there is at least one α for which the second term is positive. Thus $v(f) > 0$, which implies that $df_p(v) = (vf) d/dt\big|_{f(p)} \neq 0$, where t is the standard coordinate on $\mathbb{R}$.

(7/15/15) Page 120, proof of Proposition 5.46: At the beginning of the proof, insert this sentence: "Let $F: D \hookrightarrow M$ denote the inclusion map."

(7/21/18) Page 121, line 5: Change x^m to x^n.

(9/15/19) Page 123, Problem 5-6: Add the assumption that $m > 0$.

(7/8/19) Page 124: At the end of the page, add a new problem:
5-24. Suppose M is a smooth manifold with boundary, N is a smooth manifold, and $F: N \to M$ is a smooth map whose image is contained in ∂M. Show that F is smooth as a map into ∂M, and use this to prove that ∂M has a unique smooth structure making it an embedded submanifold of M.

(12/19/18) Page 129, proof of Sard's theorem, second paragraph: Just before the last sentence of the paragraph, insert the following: "In the $\mathbb{H}^n$ case, extend F to a smooth map on an open subset of $\mathbb{R}^m$, and replace U by that open subset; if we can show that the set of critical values of the extended map has measure zero, then the same is true of the set of critical values of F."

(3/16/19) Page 129, displayed equation near the bottom of the page: Change "ith partial derivatives" to "ith-order partial derivatives."

(12/26/18) Page 130, just below equation (6.2): Right after the displayed equation, insert "(where the component functions $F^2, \dots, F^n$ might be different from the ones in the original coordinate chart)."

(3/28/20) Page 131, two lines below the first displayed equation: Change $A'(R/K)^{k+1}$ to $A'(R\sqrt{m}/K)^{k+1}$.

(1/8/18) Page 131, three lines below the first displayed equation: Insert "at most" before "K^m balls."

(3/28/20) Page 131, second displayed equation: Change the left-hand side to $K^m(A')^n(R\sqrt{m}/K)^{n(k+1)}$, and in the next line, change the definition of A'' to $A'' = (A')^n(R\sqrt{m})^{n(k+1)}$.

(4/17/13) Page 132, proof of Lemma 6.13, second paragraph: This argument does not apply when $\partial M \neq \varnothing$, because in that case $M \times M$ is not a smooth manifold with boundary. Instead, we can consider the restrictions of κ to $(M \times \operatorname{Int} M) \smallsetminus \Delta_M$ and to $(M \times \partial M) \smallsetminus \Delta_M$ (both of which are smooth manifolds with boundary), and note that there is a point $[v] \in \mathbb{RP}^{N-1}$ that is not in the image of τ or either of these restrictions of κ. [Thanks to David Iglesias Ponte for suggesting this correction.]

(10/25/21) Page 134, proof of Theorem 6.15, just after the fourth paragraph of the proof: Insert the following: "In case M is an arbitrary compact subset of a larger manifold $\widetilde{M}$ with or without boundary, we can adapt this argument to obtain an embedding of a neighborhood of M into $\mathbb{R}^{nm+m}$. After covering M with finitely many regular coordinate balls or half-balls for $\widetilde{M}$, the argument above produces an injective immersion $F\colon \bigcup_i \bar{B}_i \to \mathbb{R}^{nm+m}$, which is an embedding because its domain is compact; the restriction of this map to the union of the sets B_i is the desired embedding." [This is needed in the ensuing argument for the noncompact case, because the sets E_i might not be regular domains when $\partial M \neq \varnothing$.]

(7/3/15) Page 134, displayed equations two-thirds of the way down the page: In the definition of E_i, there's an "$i - i$" that should be "$i - 1$." It should read $E_i = f^{-1}\big([b_{i-1}, a_{i+1}]\big)$.

(10/24/21) Page 134, just below the displayed equations two-thirds of the way down the page: Delete the sentence "By Proposition 5.47, each E_i is a compact regular domain." Two lines below that, replace "smooth embedding of E_i" with "smooth embedding of a neighborhood of E_i."

(7/2/18) Page 137, first paragraph under the subheading "Tubular Neighborhoods," fifth line: Change R^n to $\mathbb{R}^n$.

(7/27/18) Page 138, proof of Theorem 6.23, end of the first paragraph: Change "standard coordinate frame" to "standard coordinate basis."

(11/25/12) Page 145, statement of Corollary 6.33: After "immersed submanifold," insert "with $\dim S = \dim M$."

(12/5/16) Page 145, paragraph above Prop. 6.34: In the definition of smooth family of maps, replace "$F\colon M \times S \to N$" by "$F\colon N \times S \to M$."

(9/28/19) Page 146, equation (6.9): Should read $dF\big(T_{(p,s)}W\big) \subseteq T_qX$. [Change the equal sign to subset.]

(9/28/19) Page 146, line below the last displayed equation: Change "$= T_qX$" to "$\subseteq T_qX$."

(11/25/12) Page 148, Problem 6-13: Delete part (c). [This statement is simply wrong. It is true with the added hypothesis that F' is an embedding, but then it's essentially just a restatement of part (b).]

(2/10/18) Page 150, last line: Change "Theorem 20.16" to "Theorem 20.22."

(12/30/17) Page 160, first line: Change $R_{hh_1^{-1}}$ to $R_{h_1^{-1}h}$.

(2/16/18) Page 164, just above the subheading: Replace the last line of the proof of Prop. 7.23 by "The action is smooth because each φ can be written locally as a composition of a smooth local section followed by π."

(8/26/14) Page 169, first line: Change $\widetilde{G}$ to G.

(6/21/20) Page 169, statement of Theorem 7.35: Replace the phrase "closed Lie subgroups such that N is normal" by "Lie subgroups such that N is normal and closed." [In fact, using the result of Theorem 19.25 later in the book, the hypothesis that N is closed can also be omitted.]

(3/18/19) Page 171, third line from the end of the proof: Change E_i to E_j, so the formula reads $\rho^i_j(g) = \pi^i(g \cdot E_j)$.

(9/17/14) Page 173, Problem 7-21: Replace the first sentence by "Prove that the groups in Problem 7-20 are isomorphic to direct products of the indicated groups in cases (a) and (c) if and only if n is odd, and in cases (b) and (d) if and only if $n = 1$."

(1/18/21) Page 178, Example 8.10(d): Change "Example 8.4" to "Example 8.5."

(9/15/23) Page 179, statement of Lemma 8.13: Change "local frame for $T\mathbb{R}^n$" to "local frame for $\mathbb{R}^n$."

(6/9/19) Page 184, Example 8.20, next-to-last line: Change $p = (u, v)$ to $q = (u, v)$.

(3/19/21) Page 184, proof of Proposition 8.22: After "Proposition 5.37," insert "in the case $\partial S = \varnothing$. When S has nonempty boundary, the proof of Proposition 5.37 still goes through using boundary slice coordinates for S."

(11/17/12) Page 196, proof of Proposition 8.45, next-to-last line: Should read "$F_* \circ \left(F^{-1}\right)_* = \left(F \circ F^{-1}\right)_* = \mathrm{Id}_{\mathrm{Lie}(H)}$ and $\left(F^{-1}\right)_* \circ F_* = \mathrm{Id}_{\mathrm{Lie}(G)}$."

(4/6/24) Page 197, first paragraph: Change "proposition" to "theorem."

(4/6/24) Page 197, paragraph following the proof of Theorem 8.46: Change "proposition" to "theorem" (twice).

(5/27/17) Page 208, first line: Change to "This is just the existence and smoothness statements of Theorem D.1 ...."

(4/6/24) Page 213, line 6 of the proof: Change "to same ODE" to "to the same ODE."

(3/10/16) Page 213, first sentence of the last paragraph: The definition of t_0 should be $t_0 = \sup\{t \in \mathbb{R} : (t, p_0) \in W\}$.

(5/24/19) Page 214, Fig. 9.6: The shaded area should be labeled W, not $\mathcal{D}$.

(12/2/15) Page 217, Fig. 9.7: Both occurrences of φ should be Φ.

(12/2/15) Page 219, second displayed equation: Change "$V^j(0, p) = 0$" to "$\Phi^j(0, p) = 0$."

(12/2/15) Page 219, two lines below (9.12): Here and in the rest of the paragraph, change p_0 to p_1 (seven times) to avoid confusion with the prior unrelated use of p_0 in this proof.

(5/29/16) Page 222, just below the section heading: Insert the following sentence: "On a manifold with boundary, the definitions of flow domain, flow, and infinitesimal generator of a flow are exactly the same as on a manifold without boundary."

(2/15/19) Page 223, line 2: Change $\delta\colon M \to \mathbb{R}^+$ to $\delta\colon \partial M \to \mathbb{R}^+$.

(8/19/14) Page 223, proof of Theorem 9.26: There's a gap in this proof, because it is not necessarily the case that $M(a)$ is a regular domain in $\operatorname{Int} M$. To correct the problem, we have to choose our collar neighborhood more carefully. Replace the first sentence of the proof by the following paragraph:

"Theorem 9.25 shows that ∂M has a collar neighborhood C_0 in M, which is the image of a smooth embedding $E_0\colon [0,1) \times \partial M \to M$ satisfying $E_0(0, x) = x$ for all $x \in \partial M$. Let $f\colon M \to \mathbb{R}^+$ be a smooth positive exhaustion function. Note that $W = \{(t, x) : f(E_0(t, x)) > f(x) - 1\}$ is an open subset of $[0,1) \times \partial M$ containing $\{0\} \times \partial M$. Using a partition of unity as in the proof of Theorem 9.20, we may construct a smooth positive function $\delta\colon \partial M \to \mathbb{R}$ such that $(t, x) \in W$ whenever $0 \le t < \delta(x)$. Define $E\colon [0,1) \times \partial M \to M$ by $E(t, x) = E_0(t\delta(x), x)$. Then E is a diffeomorphism onto a collar neighborhood C of ∂M, and by construction $f(E(t, x)) > f(x) - 1$ for all $(t, x) \in [0,1) \times \partial M$. We will show that for each $a \in (0,1)$, the set $E([0,a] \times \partial M)$ is closed in M. Suppose p is a boundary point of $E([0,a] \times \partial M)$ in M; then there is a sequence $\{(t_i, x_i)\}$ in $[0,a] \times \partial M$ such that $E(t_i, x_i) \to p \in M$. Then $f(E(t_i, x_i))$ remains bounded, and thus $f(x_i) < f(E(t_i, x_i)) + 1$ also remains bounded. Since ∂M is closed in M, $f|_{\partial M}$ is also an exhaustion function, and therefore the sequence $\{x_i\}$ lies in some compact subset of ∂M. Passing to a subsequence, we may assume $(t_i, x_i) \to (t_0, x_0)$, and therefore $p = E(t_0, x_0) \in E([0,a] \times \partial M)$."

Then at the end of the first paragraph of the proof, add the following sentences:

"To see that $M(a)$ is a regular domain, note first that it is closed in M because it is the complement of the open set $C(a)$. Let $p \in M(a)$ be arbitrary. If $p \notin E([0,a] \times \partial M)$, then p has a neighborhood in $\operatorname{Int} M$ contained in $M(a)$ by the argument above. If $p \in E([0,a] \times \partial M)$, then $p = E(a, x)$ for some $x \in \partial M$, and C is a neighborhood of p in which $M(a) \cap C$ is the diffeomorphic image of $[a,1) \times \partial M$."

(1/30/14) Page 223, proof of Theorem 9.26, last line of the first paragraph: Change $0 \le t < a$ to $0 \le s < a$.

(1/30/14) Page 225, Example 9.31: At the end of the example, insert the sentence "If $n \ge 2$, then $M_1 \# M_2$ is connected."

(7/25/16) Page 226, Example 9.32, fifth line: Replace the sentence beginning "It is a smooth manifold without boundary ..." by "It is a topological manifold without boundary, and can be given a smooth structure such that each of the natural maps $M \to D(M)$ (induced by inclusion into the left and right summands of the disjoint union) is a smooth embedding."

(3/2/21) Page 230, line 1 and first displayed equation: Change $\theta_t(x)$ to $\theta_t(u)$ (twice).

(4/23/13) Page 230, second paragraph: "from Case" should be "from Case 1."

(2/26/18) Page 230, fourth paragraph, last line: Change $[X,Y]$ to $[V,W]$.

(9/8/18) Page 234, proof of Theorem 9.46, second paragraph: Replace the two parenthesized sentences by the following: "(To see this, just choose $\varepsilon_1 > 0$ and $U_1 \subseteq U$ such that θ_1 maps $(-\varepsilon_1, \varepsilon_1) \times U_1$ into U, and then inductively choose ε_i and U_i such that θ_i maps $(-\varepsilon_i, \varepsilon_i) \times U_i$ into U_{i-1}. Taking $\varepsilon = \min\{\varepsilon_i\}$ and $Y = U_k$ does the trick.)"

(5/29/16) Page 241, Example 9.52: At the end of the example, add the sentence "Note that u is smooth on the open set $\mathbb{R}^2 \smallsetminus \{0\}$, which is a neighborhood of S."

(6/4/14) Page 246, Problem 9-11: Delete the second sentence of the hint. [Because N is inward-pointing along ∂M, no integral curve that starts on ∂M can hit the boundary again, because the vector field would have to be tangent to ∂M or outward-pointing at the first such point.]

(11/17/21) Page 248, first displayed equation: Should read

$$V(t,p) = \left.\frac{\partial}{\partial s}\right|_{s=t} H_s\big(H_t^{-1}(p)\big).$$

(11/12/16) Page 248, Problem 9-22(c): Replace the problem statement by

(c) $\dfrac{\partial u}{\partial x} + u\dfrac{\partial u}{\partial y} = -y, \qquad u(0,y) = 0.$

[Without this sign change, the third claim in Problem 9-23 is not true.]

(11/16/20) Page 254, paragraph beginning "With respect to," third line: Replace $V_p \times \mathbb{R}^k$ with $U_\alpha \times \mathbb{R}^k$.

(11/4/21) Page 255, Example 10.8, line 5: Replace the phrase "a bijective map $\Phi|_U : \big(\pi|_S\big)^{-1}(U \cap S) \to (U \cap S) \times \mathbb{R}^k$" with "a bijective map from $\big(\pi|_S\big)^{-1}(U \cap S)$ to $(U \cap S) \times \mathbb{R}^k$." [The notation $\Phi|_U$ is inappropriate here.]

(6/17/19) Page 255, Example 10.8, lines 6–8: Replace the sentence beginning with "If E is a smooth vector bundle" by the following: "If E is a smooth vector bundle and $S \subseteq M$ is an embedded submanifold, it follows easily from the chart lemma that $E|_S$ is a smooth vector bundle. If S is merely immersed, we give $E|_S$ a topology and smooth structure making it into a smooth rank-k vector bundle over S as follows: For each $p \in S$, choose a neighborhood U of p in M over which there is a local trivialization Φ of E, and a neighborhood V of p in S that is embedded in M and contained in U. Then the restriction of Φ to $\pi^{-1}(V)$ is a bijection from $\pi^{-1}(V)$ to $V \times \mathbb{R}^k$, and we can apply the chart lemma to these bijections to yield the desired structure."

(3/30/21) Page 255, Example 10.8, last line: Change "over M" to "over S."

(11/27/20) Page 260, two lines above Proposition 10.22: Change $\tau^n(p)$ to $\tau^k(p)$.

(10/22/18) Page 261, statement of Proposition 10.25, first line: Change $\pi'\colon E \to M'$ to $\pi'\colon E' \to M'$.

(4/2/21) Page 263, first full paragraph: In the first two lines of the paragraph, change σ_1, σ_2 to τ_1, τ_2 (twice).

(7/2/14) Page 264, paragraph above the subheading, first sentence: "homomorphism" should be "homomorphisms."

(6/21/23) Page 265, proof of Lemma 10.32, fifth line: Change "basis for D_p at each point $p \in U$" to "basis for D_q at each point $q \in U$."

(4/6/24) Page 267, paragraph before Lemma 10.35: Change "proposition" to "lemma."

(8/7/23) Page 267, proof of Lemma 10.35, lines 3 & 4: Change "single slice in some coordinate ball or half-ball" to "single slice or half-slice in some coordinate ball."

(4/2/21) Page 271, Problem 10-18: Change "a properly embedded" to "an embedded."

(2/6/21) Page 271, Problem 10-19(d): Add the following: [Hint: For the "only if" direction, to show that F is compact, use a finite number of local trivializations to construct a closed set over which E is trivial.]

(2/6/22) Page 276, proof of Proposition 11.9, first line: Change "Theorem 10.4" to "Proposition 10.4."

(6/29/15) Page 278, Example 11.13, third line: Change "every coordinate frame" to "every coordinate coframe."

(6/11/19) Page 296, line 6 from the bottom: Change "closed forms" to "closed covector fields" (twice).

(4/18/20) Page 301, Problem 11-10(c): Change S^2 to $\mathbb{S}^2$.

(4/20/20) Page 301, Problem 11-13: Add the assumption that $n > 0$.

(5/19/18) Page 303, just below the commutative diagram: Insert this sentence: "A natural transformation is called a natural isomorphism if each map λ_X is an isomorphism in D."

(5/19/18) Page 303, Problem 11-18(b) and (c): Change "natural transformation" to "natural isomorphism" in both parts.

(4/7/21) Page 317, paragraph beginning "Any one": At the end of the paragraph, add this sentence: "If A and B are tensor fields, then $A \otimes B$ denotes the tensor field defined by $(A \otimes B)_p = A_p \otimes B_p$."

(5/24/18) Page 317, displayed equation just below the middle of the page: Change $A^{i_1 \dots i_k}_{j_1 \dots i_l}$ to $A^{i_1 \dots i_k}_{j_1 \dots j_l}$ on the third line of the display, and again on the line below the display. [The last lower index should be j_l, not i_l.]

(4/18/17) Page 320, statement of Proposition 12.25: Change the domain and codomain of G: It should read $G\colon P \to M$.

(4/18/17) Page 320, Proposition 12.25(e): Should read $(F \circ G)^* B = G^*(F^* B)$.

(4/17/15) Page 333, first line: Change $U \subseteq M$ to $V \subseteq M$.

(7/1/14) Page 345, Problem 13-10: In the last line of the problem statement, change $L_{\overline{g}}(\widetilde{\gamma}) > L_{\overline{g}}(\gamma)$ to $L_{\overline{g}}(\widetilde{\gamma}) \geq L_{\overline{g}}(\gamma)$, and delete the phrase "unless $\widetilde{\gamma}$ is a reparametrization of γ." [Because the definition of reparametrization that I'm using requires a diffeomorphism of the parameter domain, the original problem statement was not true.]

(12/18/12) Page 355, proof of Lemma 14.10: At the beginning of the proof, insert "Let $(E_1, \dots, E_n)$ be the basis for V dual to (ε^i)."

(12/18/12) Page 356, Case 4, second line: Should read "brings us back to Case 3."

(1/23/24) Page 357, first line after the proof of Proposition 14.11: Change "this lemma" to "this proposition."

(7/3/15) Page 368, second paragraph: At the end of the first sentence of the paragraph, insert "(see pp. 341–343)."

(7/18/17) Page 368, paragraph below equation (14.25): Change TM to $T\mathbb{R}^3$ (twice).

(9/17/14) Page 371, three lines above (14.31): Change that sentence to "The only terms in this sum that can possibly be nonzero are those for which J has no repeated indices and m is equal to one of the indices in J, say $m = j_p$."

(5/14/20) Page 374, Problem 14-2: Add "[Hint: One way to approach this is to prove first that a k-covector ω is decomposable if and only if the map from $\mathbb{R}^n$ to $\Lambda^{k-1}(\mathbb{R}^{n*})$ given by $v \mapsto v \lrcorner \omega$ has $(n-k)$-dimensional kernel.]"

(12/2/20) Page 377, line 4: Change "is a simply" to "is simply."

(10/17/21) Page 382, proof of Proposition 15.6, second paragraph: In the first sentence of the paragraph, after "smooth chart," insert "with connected domain."

(3/9/16) Page 386, just above Proposition 15.24: After "determines an orientation on ∂M," insert "if M is oriented."

(4/24/22) Page 388, last paragraph: Change "Proposition 13.6" to "Corollary 13.8."

(7/20/17) Page 389, Exercise 15.30: Change "a local isometry" to "an orientation-preserving local isometry."

(1/25/24) Page 393, Example 15.38, next-to-last line in the first paragraph: Change $\mathrm{Aut}_\pi(E)$ to $\mathrm{Aut}_q(E)$.

(5/9/20) Page 397, Problem 15-1: At the end of the last sentence, add "when $n > 1$."

(5/14/20) Page 397, Problem 15-3: Change $\overline{\mathbb{B}}^n$ to $\overline{\mathbb{B}}^{n+1}$ (twice).

(5/28/22) Page 397, Problem 15-4: Change the first sentence to "Let θ be the flow of a smooth vector field on an oriented smooth manifold." [The stated result is true also for manifolds with boundary and for nonmaximal flows, but to prove it, one must first do a little work to generalize some of the results of Theorem 9.12 to more general flows.]

(4/26/14) Page 402, lines 2–3: There should not be a paragraph break before "and."

(3/14/16) Page 403, just after the last displayed equation: Add "(In the $\mathbb{H}^n$ case, apply Theorem C.26 to the interiors of D and E considered as subsets of $\mathbb{R}^n$.)"

(5/28/18) Page 409, line 2: Change φ_i to φ.

(6/24/18) Page 415, paragraph above Example 16.19: Change "interior charts and charts with corners" to "interior charts, boundary charts, and charts with corners."

(6/2/16) Page 416, line 3 from the bottom: Change "$\gamma(0) = p$" to "$\gamma(0) = \psi(p)$."

(9/25/19) Page 418, statement of Proposition 16.21: Delete "compact," and change "n-manifold" to "$(n+1)$-manifold."

(6/24/18) Page 419, proof of Theorem 16.25, first paragraph: Replace the second and third sentences of the paragraph by the following: "By means of smooth charts and a partition of unity, we may reduce the theorem to the cases in which $M = \mathbb{R}^n$, $M = \mathbb{H}^n$, or $M = \overline{\mathbb{R}}^n_+$. The $\mathbb{R}^n$ and $\mathbb{H}^n$ cases are treated just as before."

(9/3/23) Page 423, just above equation (16.11): Change "$\beta\colon \mathfrak{X}(M) \to \Omega^{n-1}(M)$" to "$\beta\colon TM \to \Lambda^{n-1}T^*M$."

(7/22/15) Page 424, second displayed equation: Change $\iota_S^*\beta(X)$ to $\iota_{\partial M}^*\beta(X)$.

(2/18/13) Page 426, three lines below the section heading: "cam" should be "can."

(2/11/15) Page 430, Proposition 16.38(c): This statement is wrong. Change it to "If F is smooth, then $F^*\mu$ is a continuous density on M; and if F is a local diffeomorphism, $F^*\mu$ is smooth."

(5/31/22) Page 435, Problem 16-4: Change "manifold with boundary" to "manifold with nonempty boundary."

(7/27/16) Page 439, Problem 16-23: The formula for g should be

$$g = \frac{dx^2 + dy^2}{(1-x^2-y^2)^2}.$$

(2/19/13) Page 444, two lines below equation (17.4): Change $T_{(q,s)}M$ to $T_{(q,s)}(M \times \mathbb{R})$.

(4/7/24) Page 446, last line: Change c_q to C_q.

(4/7/24) Page 447, line 2: After "inclusion map," insert "and $c_q\colon M \to \{q\}$ denotes the constant map."

(6/6/18) Page 447, Corollary 17.15: Change "every closed form is exact" to "every closed p-form is exact for $p \geq 1$."

(5/15/15) Page 450, proof of Theorem 17.21, line 5: Change $H^1_{\text{dR}}(\mathbb{S}^n)$ to $H^1_{\text{dR}}(\mathbb{S}^1)$.

(8/14/17) Page 451, proof of Corollary 17.25, next-to-last line: Change $\text{Id}_{H^{n-1}_{\text{dR}}}(S)$ to $\text{Id}_{H^{n-1}_{\text{dR}}(S)}$.

(11/24/17) Pages 455–456, Proof of Theorem 17.32: The proof given in the book is incorrect, because the V_i's might not be connected, so Theorem 17.30 does not apply to them. Here's a corrected proof.

Lemma. If M is a noncompact connected manifold, there is a countable, locally finite open cover $\{V_j\}_{j=1}^\infty$ of M such that each V_i is connected and precompact, and for each j, there exists $k > j$ such that $V_j \cap V_k \neq \varnothing$.

Proof. Let $\{W_j\}_{j=1}^\infty$ be a countably infinite, locally finite cover of M by precompact, connected open sets (such a cover exists by Prop. 1.19 and Thm. 1.15). By successively deleting unneeded sets and renumbering, we can ensure that no W_j is contained in the union of the other W_i's.

Let $Y_1 = \bigcup_{i=2}^\infty W_i$. Because M is connected, each component of Y_1 meets W_1, and by local finiteness of $\{W_j\}$, there are only finitely many such components. Such a component is precompact in M if and only if it is a union of finitely many W_i's. Let V_1 be the union of W_1 together with all of the precompact components of Y_1, and let X_1 be the union of all W_i's not contained in V_1. Then V_1 is connected and precompact, and X_1 has no precompact components. Proceeding by induction, suppose we have defined connected, precompact open sets $V_1, \dots, V_m$ whose union contains $W_1 \cup \dots \cup W_m$, and such that the union X_m of all the W_i's not contained in $V_1 \cup \dots \cup V_m$ has no precompact components. Let j_m be the smallest index such that W_{j_m} is not contained in $V_1 \cup \dots \cup V_m$, and let Y_{m+1} be the union of all W_i's other than W_{j_m} not contained in $V_1 \cup \dots \cup V_m$. Any precompact component of Y_{m+1} must meet W_{j_m}, because otherwise, it would be a precompact component of X_m. Let V_{m+1} be the union of W_{j_m} with all of the precompact components of Y_{m+1}. As before, V_{m+1} is precompact and connected, and the union X_{m+1} of the W_i's not contained in $V_1 \cup \dots \cup V_{m+1}$ has no precompact components. Then by construction, for each j, the set $X_j = \bigcup_{i>j} V_i$ has no precompact components. If some V_j does not meet V_k for any $k > j$, then V_j itself is a precompact component of X_{j-1}, which is a contradiction. Thus for each j, there is some $k > j$ such that $V_j \cap V_k \neq \varnothing$. □

Proof of Theorem 17.32. Choose an orientation on M. Let $\{V_j\}_{j=1}^\infty$ be an open cover of M satisfying the conclusions of the preceding lemma. For each j, let $K(j)$ denote the least integer $k > j$ such that $V_j \cap V_k \neq \varnothing$, and let θ_j be an n-form compactly supported in $V_j \cap V_{K(j)}$ whose integral is 1. Let $\{\psi_j\}_{j=1}^\infty$ be a smooth partition of unity subordinate to $\{V_j\}_{j=1}^\infty$.

Now suppose ω is any n-form on M, and let $\omega_j = \psi_j \omega$ for each j. Let $c_1 = \int_{V_1} \omega_1$, so that $\omega_1 - c_1\theta_1$ is compactly supported in V_1 and has zero integral. It follows from Theorem 17.30 that there exists $\eta_1 \in \Omega_c^{n-1}(V_1)$ such that $d\eta_1 = \omega_1 - c_1\theta_1$. Suppose by induction that we have found $\eta_1, \dots, \eta_m$ and constants $c_1, \dots, c_m$ such that for each $j = 1, \dots, m$, $\eta_j \in \Omega_c^{n-1}(V_j)$ and

$$d\eta_j = \left(\omega_j + \sum_{i:K(i)=j} c_i \theta_i\right) - c_j \theta_j. \qquad (*)$$

Let

$$c_{j+1} = \int_{V_{j+1}} \left(\omega_{j+1} + \sum_{i:K(i)=j+1} c_i \theta_i\right).$$

Then by Theorem 17.30, there exists $\eta_{j+1} \in \Omega_c^{n-1}(V_{j+1})$ satisfying the analog of $(*)$ with j replaced by $j+1$. Set $\eta = \sum_{j=1}^\infty \eta_j$, with each η_j extended to be zero on $M \smallsetminus V_j$. By local finiteness, this is a smooth $(n-1)$-form on M. It satisfies

$$d\eta = \omega + \sum_{j=1}^\infty \left(\sum_{i:K(i)=j} c_i \theta_i\right) - \sum_{j=1}^\infty c_j \theta_j.$$

Each term $c_i\theta_i$ appears exactly once in the first sum above, so the two sums cancel each other. □

(7/27/16) Page 457, line below the second displayed equation: Change "Theorem 17.31" to "Theorem 17.30."

(7/12/16) Page 463, line above equation (17.15): Insert missing space before "Similarly."

(7/13/16) Page 464, end of proof of Corollary 17.42: Insert "Note that this construction produces a form σ whose support is contained in $U \cap V$." [This might be useful for solving Problem 18-6.]

(7/12/16) Page 471, last paragraph: Replace the sentence starting "The hardest part …" with "The hardest part is showing that the singular chain complex of M can be replaced by a chain complex built out of simplices whose images lie in either U or V, without changing the homology."

(9/12/17) Page 487, Problem 18-1, first line: Change "an oriented smooth manifold" to "a smooth manifold."

(8/8/18) Page 489, Problem 18-7(b): Add to the hint: "In order to use Lemma 17.27, you'll need to prove the following fact: Every bounded convex open subset of $\mathbb{R}^n$ is diffeomorphic to $\mathbb{R}^n$. To prove this, let U be such a subset, and without loss of generality assume $0 \in U$. First show that there exists a smooth nonnegative function $f \in C^\infty(U)$ such that $f(0) = 0$ and $f(x) \geq 1/d(x)$ away from a small neighborhood of 0, where $d(x)$ is the distance from x to ∂U. Next, show that $g(x) = 1 + \int_0^1 t^{-1} f(tx)\,dt$ is a smooth positive exhaustion function on U that is nondecreasing along each ray starting at 0. Finally, show that the map $F\colon U \to \mathbb{R}^n$ given by $F(x) = g(x)x$ is a bijective local diffeomorphism. Also, you may use the fact that the conclusion of the five lemma is still true even if the appropriate diagram commutes only up to sign."

(1/15/13) Page 491, Example 19.1(c): Delete the word "unit."

(5/22/15) Page 492, line above Proposition 19.2: Change "lie" to "Lie."

(12/17/15) Page 492, proof of Proposition 19.2, fourth line: Change "Given $p \in M$" to "Given $p \in U$."

(9/12/16) Page 506, Lemma 19.24, last line: Before "left-invariant," insert "smooth."

(6/1/20) Page 512, Problem 19-4: In the first line of the problem, change "all three coordinates are positive" to "z is positive." Then replace the last sentence by "Find an explicit global chart on U in which D is spanned by the first two coordinate vector fields." [Technically it might not be a flat chart because its image need not be a cube in $\mathbb{R}^3$.]

(7/27/22) Page 513, Problem 19-10: Add the following to the end of the problem statement: "(Transversality to an immersed submanifold is defined exactly as in the embedded case.)"

(10/4/17) Page 518, sentence before Prop. 20.3: Change "one-parameter subgroups of $\mathrm{GL}(n,\mathbb{R})$" to "one-parameter subgroups of subgroups of $\mathrm{GL}(n,\mathbb{R})$."

(5/23/16) Page 521, first displayed equation: Change $d\Phi_0$ to $d\Phi_e$ (twice).

(7/10/23) Page 524, first paragraph, last line: Change "$U_i \subseteq U_0$ and $\widetilde{U}_i \subseteq \widetilde{U}_0$" to "$U_i \subseteq U_0$, $V_i \subseteq \Phi(\widetilde{U}_0)$, and $\widetilde{U}_i \cap \mathfrak{h} \subseteq U_0$."

(6/9/19) Page 528, line 9: Change two instances of (g,p) in subscripts to (g,q).

(5/19/18) Page 528, just below the displayed equation in the middle of the page: The smoothness of the map σ_q is not quite immediate from the definition. Replace the three sentences beginning "It follows" with this: " Because S_p is a weakly embedded submanifold by Theorem 19.17, to show that σ_q is a smooth local section of S_p, it suffices to show that it is smooth into $G \times M$ and takes its values in S_p. The first component function is smooth as a map into G by smoothness of group multiplication. To show that the second component is smooth into M as a function of $\widehat{X}$ (and therefore of $\exp X$), you need to use the argument sketched out just below equation (20.10): as in the proof of Prop. 20.8, apply the fundamental theorem on flows to the vector field $\Xi_{(p,X)} = \left(\widehat{X}_g, 0\right)$ on $M \times \mathfrak{g}$. A straightforward computation shows that $\gamma(t) = \left(g \exp tX, \eta_{(\widehat{X})}(t,q)\right)$ is an integral curve of $\widetilde{X}$ starting at (g,q), from which it follows easily that $\sigma_q(g \exp X) = \gamma(1) \in S_p$."

(1/10/17) Page 537, Problem 20-6(a): Change $B \in \mathfrak{gl}(n,\mathbb{R})$ to $B \in \mathfrak{sl}(n,\mathbb{R})$.

(5/31/16) Page 538, Problem 20-11(b): Here's a better hint, which doesn't require proving part (a) first: "[Hint: Consider the graph of F as a subgroup of $G \times H$.]"

(10/18/17) Page 542, middle of the paragraph before Example 21.3: Change "the action of $\mathbb{R}^k$ on $\mathbb{R}^n$" to "the action of $\mathbb{R}^k$ on $\mathbb{R}^k \times \mathbb{R}^n$."

(12/28/23) Page 543, 6th line from the bottom: Change "subsequence of G_K" to "sequence in G_K."

(2/25/18) Page 548, last two lines: Allen Hatcher's name is misspelled. (Sorry, Allen.)

(5/23/16) Page 549, proof of Proposition 21.12, last sentence: Change the first phrase of that sentence to "Second, if $p, p' \in E$ are in different orbits and $\pi(p) \neq \pi(p')$, ..." Then add the following sentences at the end of the proof: "If p and p' are in different orbits and $\pi(p) = \pi(p')$, let W be an evenly covered neighborhood of $\pi(p)$, and let V, V' be the components of $\pi^{-1}(W)$ containing p and p', respectively. For any $g \in \mathrm{Aut}_\pi(E)$, a simple connectedness argument shows that $g \cdot V$ is a component of $\pi^{-1}(W)$; if it had nontrivial intersection with V it would have to be equal to V, which would imply $g \cdot p = p'$, a contradiction."

(7/26/16) Page 567, two lines above Proposition 22.8: Insert "a" before "2-covector."

(10/9/15) Page 568, Example 22.9(a), first line: The coordinates should be $\left(x^1, \dots, x^n, y^1, \dots, y^n\right)$. (The last coordinate is y^n, not x^n.)

(11/17/21) Page 571, line below equation (22.5): Delete the spurious word "theorem" at the end of the line.

(3/27/19) Page 572, middle of the page: Replace the sentence starting "On the other hand" by this: "On the other hand, the left-hand side is just the ordinary t-derivative of a time-dependent tensor on a fixed vector space, and expanding in terms of a basis shows that it satisfies a similar product rule:"

(10/5/17) Page 573, statement of Proposition 22.15, second line: Change "$V\colon J\times M$" to "$V\colon J\times M\to TM$"; and change ψ to θ.

(11/18/17) Page 583, line 4: Change $\mathbb{R}^{2n+1}\smallsetminus\{0\}$ to $\mathbb{R}^{2n+2}\smallsetminus\{0\}$.

(7/26/16) Page 583, third displayed equation: Should read

$$T \lrcorner\, d\Theta = -2\sum_{i=1}^{n+1}\left(x^i\,dx^i + y^i\,dy^i\right) = -d\left(|x|^2+|y|^2\right).$$

(7/26/16) Page 583, two lines below the third displayed equation: The formula for $d\Theta(N,T)$ should be $d\Theta(N,T) = 2\left(|x|^2+|y|^2\right)$.

(11/28/12) Page 584, Exercise 22.29: Part (b) should read

$$\text{(b)}\quad T = \frac{\partial}{\partial z};$$

(8/14/14) Page 584, paragraph above Theorem 22.33: Change all occurrences of θ in this paragraph to ψ, to avoid confusion with the use of θ for a contact form elsewhere in this section.

(11/24/17) Page 585, statement of Theorem 22.34, last line: Change H to F.

(11/17/12) Page 587, equation (22.27): Change both occurrences of $\sigma(s)$ to $\sigma(x)$.

(6/7/22) Page 591, Problem 22-5: Add the hypothesis $n>0$.

(11/18/17) Page 592, Problem 22-15: Add the hypothesis that M is connected.

(9/22/15) Page 608, Proposition A.41(a): Insert the following phrase at the beginning of this statement: With the exception of the word "closed" in part (d).

(7/22/13) Page 616, Proposition A.77(b), last line: Change $\widetilde{f}(0)$ to $\widetilde{f}_e(0)$.

(12/19/18) Page 619, proof of Lemma B.2, fourth line: Replace "By Exercise B.1(b)" with "If w_1 is equal to one of the v_i's, then the ordered $(n+1)$-tuple $(w_1,v_1,\dots,v_n)$ is linearly dependent; if not, then by Exercise B.1(b), …."

(9/1/16) Page 632, Exercise B.29: Change "by a matrix" to "by a certain matrix" (twice).

(12/19/18) Page 637, Exercise B.42: Delete the words "is a homeomorphism that." [Checking that it's a homeomorphism requires the norm topology, which is not defined until later on that page.]

(9/6/16) Page 637, Exercise B.44: Change "basis map" to "basis isomorphism."

(12/19/18) Page 653, proof of Proposition C.21, second paragraph, second line: Change f to f_D.

(2/25/18) Page 658, two lines above (C.15): Change $B_\delta(0)$ to $\overline{B}_\delta(0)$.

(2/25/18) Page 660, display (C.20): Change $F^{-1}(x)$ to $F^{-1}(y)$.

(1/18/21) Page 664, statement of Theorem D.1(b): After the phrase "Any two differentiable solutions to (D.3)–(D.4)," insert "defined on intervals containing t_0."

(12/2/15) Page 666, just below the fifth display: After the sentence ending "by our choice of δ and ε," insert "(If $t < t_0$, interchange t and t_0 in the second line above.)"

(1/18/21) Page 667, statement of Theorem D.4: After the phrase "any two differentiable solutions to (D.3)–(D.4)," insert "defined on intervals containing t_0."

(2/13/24) Page 667, last paragraph: Change U to U_0 (twice).

(2/13/24) Page 668, line 2: Change W to $\overline{W}$.

(2/13/24) Page 668, paragraph below equation (D.10): In the fourth line of the paragraph, change $\overline{W}$ to W; and in the fifth line, change W to $\overline{W}$.

(1/18/21) Page 670, displayed inequality between (D.17) and (D.18): Change n to n^2.

(1/18/21) Page 670, last line: Change n to n^2 in the definition of B.

(1/18/21) Page 671, inequality (D.19): Change n to n^2 (twice).

(12/15/20) Page 671, just below (D.19): Replace the sentence "Since the expression on the right can be made as small as desired by choosing h and $\widetilde{h}$ sufficiently small, this shows …" by the following: "Thus the expression on the left can be made as small as desired by choosing h and $\widetilde{h}$ sufficiently small. This shows …"

(6/11/19) Page 692: Under the entry for "Form," delete the references to page 294 for "closed" and page 292 for "exact."

(2/25/18) Page 693: The index entry for "Hatcher, Allen" is misspelled.