CORRECTIONS TO
Introduction to Topological Manifolds
(Second Edition)
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(2/14/15) Page 23, Exercise 2.6, first line: Change “collection of topologies” to “nonempty collection of topologies.”

(5/17/12) Page 37, three lines from the bottom: Change “Exercise 2.49” to “Example 2.49.”

(12/7/15) Page 104, proof of Proposition 4.60: Change “bump function we seek.”

(1/20/11) Page 74, line 5: Change this statement to read “Let and is not a disjoint union.”

(8/23/11) Page 76, last paragraph of the proof of Theorem 3.79: In the second line of the paragraph, change “embedding of $N$” to “embedding of $M$.” In the fourth line, change “embedding of $M$” to “embedding of $N$.”

(3/23/12) Page 67, Example 3.52, second sentence: Change this sentence to read “Let be the equivalence relation on $X$ such that $a_1 \sim a_2$ for all $a_1, a_2 \in A$ and $x \sim x$ for all other $x \in X$; the partition . . . .”

(5/17/12) Page 67, Example 3.53, last line: Change $B^n$ to $B^{n+1}$.

(10/7/11) Page 74, line 5: Change this statement to read “As a set, $X \cup_f Y$ is the disjoint union . . . .” [The topology on $X \cup_f Y$ is not the disjoint union topology.]

(7/16/11) Page 76, last paragraph of the proof of Theorem 3.79: In the second line of the paragraph, change “embedding of $N$” to “embedding of $M$.” In the fourth line, change “embedding of $M$” to “embedding of $N$.”

(8/23/11) Page 67, Example 4.4.: Insert another “if” after “if and only.”

(8/23/11) Page 88, proof of Proposition 4.9, fourth paragraph: In the first sentence of that paragraph, change “open subsets of $\bigcup_{a \in A} B_a$” to “open subsets of $X$ whose union contains $\bigcup_{a \in A} B_a$.”

(8/23/11) Page 97, line 10: Change $B_{n_{\text{min}}}(x)$ to $B_{n_{\text{max}}}(x)$.

(3/9/11) Page 98, line 3 from bottom: Change “this proposition” to “this lemma.”

(11/11/13) Page 104, proof of Proposition 4.60: At the end of the first paragraph, change $B_{2r(x)}(x) \subseteq \hat{U}_1$ to $B_{2r(x)}(x) \subseteq \hat{U}_1$. [Without this change, it might not be the case that $\mathcal{B}$ covers $M$.]

(8/23/11) Page 106, line 3 from the bottom: Change “countable union” to “countable intersection.”

(8/23/11) Page 109, statement of Lemma 4.74: Insert another “if” after “if and only.”

(7/19/15) Page 110, next-to-last line: Change $M$ to $X$.

(8/23/11) Page 114, proof of Corollary 4.83: This proof is incorrect. Replace it with the following: “Given a closed subset $A \subseteq X$ and a neighborhood $U$ of $A$, Lemma 4.80 shows that there is a neighborhood $V$ of $A$ such that $V \subseteq U$. By Urysohn’s lemma, there exists a continuous function $f: X \to [0, 1]$ such that $f \equiv 1$ on $A$ and $f \equiv 0$ on $X \setminus V$. This function satisfies $\sup f \leq V \subseteq U$, so it is the bump function we seek.”

(8/23/11) Page 121, proof of Lemma 4.94: Replace the last two sentences of the proof with the following: “Thus $x$ lies in the closure of $A \cap K$ in $K$. Because $A \cap K$ is closed in $K$, it follows that $x \in A \cap K \subseteq A$.”

(8/23/11) Page 123, Problem 4-15(d): Change “every connected neighborhood” to “every neighborhood.”

(9/16/11) Page 126, Problem 4-30: Change $\{A_a\}$ to $\{X_a\}_{a \in A}$.

(1/20/11) Page 133, proof of Proposition 5.7: This should refer to Problem 5.8, not 5.7.

(5/17/12) Page 136, four lines below the displayed equations: Change “both $X_n'$ and $X_n''$ are open” to “both $X_n'$ and $X_n''$ are open.”

(1/20/11) Page 137, statement of Lemma 5.13: Change “discrete” to “closed and discrete.”

(1/20/11) Page 137, proof of Lemma 5.13, first paragraph: In line 1, change “discrete” to “closed and discrete”; and in line 2, change “discrete subset” to “closed discrete subset.”

(1/20/11) Page 137, proof of Theorem 5.14, second paragraph: Change “infinite discrete subset” to “infinite closed discrete subset.”
(9/16/11) Page 141, line 5 from the bottom: Change $\tilde{U}^n_{\alpha}$ to $\tilde{U}^{n+1}_{\alpha}$ (twice).

(2/5/13) Page 141, line 4 from the bottom: Change “the minimum” to “one-half the minimum.”

(2/5/13) Page 141, line 3 from the bottom: Change “supported in $\partial D^n_y + (\epsilon/2)$” to “supported in $D^n_y \cup \partial D^{n+1}_y + (\epsilon/2)$”

(3/24/11) Page 156, Problem 5-4: add the hypothesis that $\dim M > 1$.

(9/16/11) Page 172, first paragraph, next-to-last line: Change $P_1 \sqcup Q$ to $P_1 \sqcup Q$.

(9/16/11) Page 176, Fig. 6.21: The label $b$ near the lower right should be $c$, and the label $w$ near the middle of the right-hand side should be $x$.

(9/16/11) Page 190, line 3 from the bottom: Change $\Phi_g(f)$ to $\Phi_g[f]$.

(11/20/11) Page 193, proof of Proposition 7.16, second paragraph, line 2: Change “$H_1 = f$” to “$H_1 = \tilde{f}$.”

(11/25/12) Page 211, line 6: Delete redundant “each.”

(7/9/15) Page 215, Problem 7-9: Change “connected” to “path-connected.”

(1/20/11) Page 224, two lines above the subheading: Change $\tilde{f}_0(1)$ to $\tilde{f}_1(0)$.

(1/20/11) Page 228, displayed equations (8.4): Replace these equations by

\[
\deg \varphi = \deg (\rho_{\varphi} \circ \varphi)_*, \\
\deg \psi = \deg (\rho_{\varphi} \circ \psi)_*.
\] (8.4)

(7/13/15) Page 230, Problem 8-5: Replace the last sentence of the hint by the following: “Prove that $p_n|_{S^1}$ and $p_n(z) = z^n$ are homotopic as maps from $S^1$ to $\mathbb{C} \setminus \{0\}$. If $p$ has no zeros, use degree theory to derive a contradiction.”

(7/13/15) Page 231, Problem 8-10(c): Change “index of $V$ around the loop $\omega$” to “winding number of $V$ around the loop $\omega$.”

(9/16/11) Page 239, fourth line below the section heading: Change “generated by $G$” to “generated by $S$.”

(9/16/11) Page 263, line 2: Change $\tilde{U} \cap \tilde{V}$ to $q(D \setminus \{z\})$.

(8/2/13) Page 268, lines 2 & 3: Change “preceeding corollary” to “preceeding theorem.”

(5/23/11) Page 269, line below equation (10.7): Insert missing comma after “surjective.”

(7/8/14) Page 275, Problem 10-21(c): Delete “with nonempty intersection.”

(5/17/12) Page 302, Problem 11-5, first line: Change “dimension $n$” to “dimension $n \geq 2$.”

(12/10/15) Page 303, Problem 11-12(c): Change “$(1,0)$ or $(-1,0)$” to “1 or $-1$” [to be consistent with the complex notation used elsewhere for $S^1$].

(5/17/12) Page 305, Problem 11-20: At the end of the problem, add: “For the counterexample, you may use without proof the fact that $S^2$ is not contractible. (This follows, for example, from Corollary 13.11 and Theorem 13.23.)”

(7/8/14) Page 315, paragraph above the displayed diagram: After “$Q$ is a normal covering map,” insert “and $\tilde{H} = Aut_Q(E)$.”

(7/8/14) Page 315, just below the displayed diagram: Replace the last two paragraphs on page 315 and the first (partial) paragraph on page 316 with the following:

We have to show that $\tilde{q}$ is a covering map. Let $x \in X$ be arbitrary, and let $U$ be a neighborhood of $x$ that is evenly covered by $q$. We will show that $U$ is also evenly covered by $\tilde{q}$. Given a component $U_i$ of $q^{-1}(U)$, let $\tilde{U}_i = Q(U_i) \subseteq \tilde{E}$; then $\tilde{U}_i$ is connected, and it is open in $\tilde{E}$ because $Q$ is an open map (Proposition 11.1). Suppose $\tilde{U}_i = Q(U_i)$ and $\tilde{U}_j = Q(U_j)$ are any two such sets. If they have a point $\tilde{e}$ in common, then $\tilde{e} = Q(e_i) = Q(e_j)$ for some $e_i \in U_i$ and $e_j \in U_j$. Since $Q$ identifies points of $E$ if and only if they are in the same $\tilde{H}$-orbit, there is some $\varphi \in \tilde{H}$ such that $e_j = \varphi(e_i)$. Then $\varphi(U_i) = U_j$ by Proposition 12.1(c), so $Q(U_i) = Q \circ \varphi(U_i) = Q(U_j)$. This shows that any such sets $\tilde{U}_i, \tilde{U}_j$ are either disjoint or equal. Since $Q$ is surjective, $\tilde{q}^{-1}(U)$ is equal to the disjoint union of the connected open sets $\tilde{U}_i$ as $U_i$ ranges over the components of $q^{-1}(U)$. 
It remains only to show that for any such set $\hat{U}_i$, the restricted map $\hat{q}: \hat{U}_i \to U$ is a homeomorphism. The following diagram commutes:

$$
\begin{array}{ccc}
U_i & \overset{Q}{\longrightarrow} & \hat{U}_i \\
\downarrow{q} & & \downarrow{\hat{q}} \\
U & \end{array}
$$

(12.3)

Since $q = \hat{q} \circ Q$ is injective on $U_i$, so is $Q$; and $Q: U_i \to \hat{U}_i$ is surjective by definition. Because $Q$ is an open map, it follows that $Q: U_i \to \hat{U}_i$ is a homeomorphism. Since $q$ and $Q$ are homeomorphisms in (12.3), so is $\hat{q}$.

(9/27/11) Page 318, statement of Proposition 12.21, second line: Insert “on” after “acting.”

(9/23/14) Page 321, line 4: Change $E \times E$ to $E$.

(9/27/11) Page 329, paragraph just below the diagram: Change every occurrence of $\bar{p}$ to $\bar{q}$ (five times).

(9/27/11) Page 330, just below the bulleted list: Change $\bar{p}$ to $\bar{q}$.

(9/27/11) Page 332, first full paragraph, second line: Change $\bar{p}$ to $\bar{q}$.

(9/27/11) Page 332, second full paragraph, lines 6 and 7: Change $\bar{p}$ to $\bar{q}$ (twice).

(9/16/14) Page 335, Problem 12-10: Interchange the definitions of $G$ and $H$ in the sixth and seventh lines. (Otherwise, part (c) is false as stated.)

(10/12/14) Page 337, Problem 12-19: Replace the first sentence of the problem with the following: “Suppose we are given a continuous action of a metrizable topological group (e.g., a discrete group) $G$ on a first countable Hausdorff space $E$.”

(9/27/11) Page 352, lines 3 and 4: Change $c_p$ to $c_q$ (twice), and change $p$ to $q$ (twice).

(10/8/15) Page 370, line 5 from the bottom: Change “It follows . . .” to “Assuming $X$ is path-connected, it follows . . .”

(10/8/15) Page 371, at the end of the first (partial) paragraph: Insert “If $X$ is not path-connected, just apply this argument to the path component containing the image of $\varphi$, and use Proposition 13.5.”