Corrections to
Introduction to Topological Manifolds (First edition)
by John M. Lee
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Changes or additions made in the past twelve months are dated.

- **Page 29, statement of Lemma 2.11:** The second sentence should be replaced by "*If the open subsets of X are exactly those sets that satisfy the basis criterion with respect to $\mathcal{B}$, then $\mathcal{B}$ is a basis for the topology of X.*"
- **Page 29, paragraph before Exercise 2.15:** Instead of "the topologies of Exercise 2.1," it should say "some of the topologies of Exercise 2.1."
- **Page 30, last sentence of the proof of Lemma 2.12:** Replace U by $f^{-1}(U)$ (three times).
- **Page 30, first paragraph in the "Manifolds" section:** Delete the sentence "Let X be a topological space."
- **Page 38, Problem 2-16(b):** Replace part (b) by "Show that for any space Y, a map $f\colon X \to Y$ is continuous if and only if $p_n \to p$ in X implies $f(p_n) \to f(p)$ in Y."
- **Page 38, Problem 2-18:** This problem should be moved to Chapter 3, because $\operatorname{Int} M$ and ∂M are to be interpreted as having the subspace topologies. Also, for this problem, you may use without proof the fact that $\operatorname{Int} M$ and ∂M are disjoint.
- **Page 40, last line of Example 3.1:** Replace "subspace topology on B" by "subspace topology on C."

(12/7/15) **Page 46, second display:** Replace k by $k/2$ (twice) and l by $l/2$ (twice). [The tangent and cotangent functions have period $\pi/2$, not π.]

- **Page 51, proof of Proposition 3.13, third line:** $f_1(U_1), \dots, f_k(U_k)$ should be replaced by $f_1^{-1}(U_1), \dots, f_k^{-1}(U_k)$.
- **Page 51, proof of Proposition 3.14, last sentence:** Replace "the preceding lemma" by "the preceding proposition."
- **Page 52, first paragraph after Exercise 3.8:** In the first sentence, replace the words "surjective and continuous" by "surjective." Also, add the following sentence at the end of the paragraph: "It is immediate from the definition that every quotient map is continuous."
- **Page 52, last paragraph:** Change the word "quotient" to "surjective" in the first sentence of the paragraph.
- **Page 53, line 1:** Change the word "quotient" to "surjective" at the top of the page.
- **Page 53, Lemma 3.17:** Add the following sentence at the end of the statement of the lemma: (*More precisely, if $U \subset X$ is a saturated open or closed set, then $\pi|_U\colon U \to \pi(U)$ is a quotient map.*)
- **Page 82, line 3 from bottom:** Delete "$= \overline{U} \cap Z$" from the sentence beginning "Since $\overline{U \cap Z}$ ...."
- **Page 83, Example 4.30(a):** In the first sentence, change "closed" to "open" and change $\overline{B}_\varepsilon(x)$ to $B_\varepsilon(x)$.

- **Page 85, statement of Corollary 4.34:** "*countable collection*" should read "*countable union.*"
- **Page 99, Lemma 5.4:** Replace part (d) by
  (*d*) *For any topological space Y, a map $F\colon |\mathcal{K}| \to Y$ is continuous if and only if its restriction to $|\sigma|$ is continuous for each $\sigma \in \mathcal{K}$.*
- **Page 103, Proposition 5.11:** In the statement of the proposition, change "*simplicial complex*" to "1-*dimensional simplicial complex.*"
- **Page 106, line 3 from bottom:** Replace "even" by "odd."
- **Page 111, Figure 5.12:** In $S(SK)$, the points inside the small triangles should be at the intersections of the three medians.
- **Page 114, Problem 5-2:** Replace the statement of the problem by: "Let $\mathcal{K}$ be an abstract simplicial complex. For each vertex v of $\mathcal{K}$, let $\operatorname{St} v$ (the *open star* of v) be the union of the open simplices $\operatorname{Int}|\sigma|$ as σ ranges over all simplices that have v as a vertex; and define a function $t_v\colon |\mathcal{K}| \to \mathbb{R}$ by letting $t_v(x)$ be the coefficient of v in the formal linear combination representing x.
  (a) Show that each function t_v is continuous.
  (b) Show that $\operatorname{St} v$ is a neighborhood of v, and the collection of open stars of all the vertices is an open cover of $|\mathcal{K}|$."
- **Page 114, Problem 5-3:** Delete the phrase "and locally path connected."
- **Page 120, Statement of Proposition 6.2(b):** Replace $x \in \partial\mathbb{B}^2$ by $(x, y) \in \partial\mathbb{B}^2$.
- **Page 126, Proposition 6.6:** Add the hypothesis that $n \geq 2$.
- **Page 131, Part 1 of the definition of the geometric realization:** After "sides of length 1," insert "equal angles,".
- **Page 136, line 8 from bottom:** Change the surface presentation in that line to $\langle S_1, S_2, a, b, c \mid W_1c^{-1}b^{-1}a^{-1}, abcW_2\rangle$.
- **Page 139, proof of the classification theorem:** Replace the first sentence of the proof with "Let M be the compact surface determined by the given presentation."
- **Page 140, line 14:** Change "Step 3" to "Step 2."
- **Page 149, Example 7.3:** The first line should read "Define maps $f, g\colon \mathbb{R} \to \mathbb{R}^2$ by ...."
- **Page 155, line 3:** Change $\Phi_g(f)$ to $\Phi_g[f]$.
- **Page 156, Figure 7.7:** The labels $I \times I$, F, and X should all be in math italics.
- **Page 156, Exercise 7.2:** Change the first sentence to "Let X be a path connected topological space."
- **Page 159, second line from bottom:** "induced homeomorphism" should read "induced homomorphism."
- **Page 160, Proposition 7.18:** In the statement and proof of the proposition, change $(\iota_A)*$ to $(\iota_A)_*$ three times (the asterisk should be a subscript).
- **Page 174, proof of Lemma 7.35:** In the second-to-last line of the proof, change "Theorem 3.10" to "Theorem 3.11."

- **Page 176, Problem 7-5:** Change "compact surface" to "connected compact surface."
- **Page 188, proof of Theorem 8.7:** Replace the third sentence of the proof by "If $f\colon I \to \mathbb{S}^n$ is any loop based at a point in $U \cap V$, by the Lebesgue number lemma there is an integer m such that on each subinterval $[k/m, (k+1)/m]$, f takes its values either in U or in V. If $f(k/m) = N$ for some k, then the two subintervals $[(k-1)/m, k/m]$ and $[k/m, (k+1)/m]$ must be both be mapped into V. Thus, letting $0 = a_0 < \cdots < a_l = 1$ be the points of the form k/m for which $f(a_i) \neq N$, we obtain a sequence of curve segments $f|_{[a_{i-1},a_i]}$ whose images lie either in U or in V, and for which $f(a_i) \neq N$." Also, in the last line of the proof, replace "f is homotopic to a path" by "f is path homotopic to a loop."
- **Page 189, proof of Proposition 8.9:** In the last sentence of the proof, change the domain of H to $I \times I$, and change the definition of H to

$$H(s,t) = (H_1(s,t), \ldots, H_n(s,t)).$$

- **Page 191, Problem 8-7:** In the third line of the problem, change $\varphi(\gamma)$ to $\varphi_*(\gamma)$.
- **Page 192, line 4:** Change the definition of φ to $\varphi(x) = (x - f(x))/|x - f(x)|$.
- **Page 199, second-to-last paragraph:** In the second sentence, after "a product of elements of S," insert "or their inverses."
- **Page 208, Problem 9-4(b):** Change the first phrase to "Show that $\operatorname{Ker} f_1 * f_2$ is equal to the normal closure of $\operatorname{Im} j_1 * j_2, \ldots$." Add the following hint: "[Hint: Let N denote the normal closure of $\operatorname{Im} j_1 * j_2$, so it suffices to show that $f_1 * f_2$ descends to an isomorphism from $(G_1 * G_2)/N$ to $H_1 * H_2$. Construct an inverse by showing that each composite map $G_j \hookrightarrow G_1 * G_2 \to (G_1 * G_2)/N$ passes to the quotient yielding a map $H_j \to (G_1 * G_2)/N$, and then invoking the characteristic property of the free product.]"
- **Page 213, proof of Proposition 10.5:** In the second sentence of the proof, change $\{q\}$ to $\{*\}$.
- **Page 218, Figure 10.4:** In the upper diagram, one of the arrows labeled a_i should be reversed.
- **Page 227, line 8:** Replace $\overline{R} * \overline{S}$ by $\overline{R * S}$.
- **Page 233, last line:** Change the last sentence to "This brings us to the next-to-last major subject in the book: ...."
- **Page 238, proof of Proposition 11.10, second line:** Change "p maps ..." to "f maps ...."
- **Page 248, Example 11.26:** Change $\mathcal{C}_\pi(\mathbb{P}^n)$ to $\mathcal{C}_\pi(\mathbb{S}^n)$.
- **Page 249, line 5:** Change the formula to "$p(\varphi(\widetilde{q})) = p(\widetilde{q}) = q$" (not p).
- **Page 253, Problem 11-9:** Change "path connected" to "locally path connected."
- **Page 265, Step 4:** In the second line of Step 4, replace "as in Step 3" by "as in Step 2."
- **Page 268, proof of Theorem 12.11:** The first and last paragraphs of this proof can be simplified considerably by using the result of Problem 3-15.
- **Page 272, first paragraph:** The last sentence should read "It can be identified with a quotient of the group of matrices of the form $\left(\begin{smallmatrix} \alpha & \beta \\ \overline{\beta} & \overline{\alpha} \end{smallmatrix}\right)$ (identifying two matrices if they differ by a scalar multiple), and so is a topological group acting continuously on $\mathbb{B}^2$."

- **Page 284, just below the first displayed equation:** Replace everything on that page below the first displayed equation with the following:

  We have to show that p' is a covering map. Let $q_1 \in X$ be arbitrary, and let U be a neighborhood of q_1 that is evenly covered by p. We will show that U is also evenly covered by p'. Given a component $\widetilde{U}$ of $p^{-1}(U)$, let $U' = \pi(\widetilde{U}) \subset X'$; since π is an open map (Problem 3-15), U' is open in X'. Suppose $U'_1 = \pi(\widetilde{U}_1)$ and $U'_2 = \pi(\widetilde{U}_2)$ are any two such sets. If they have a point q' in common, then $q' = \pi(\widetilde{q}_1) = \pi(\widetilde{q}_2)$ for some $\widetilde{q}_1 \in \widetilde{U}_1$ and $\widetilde{q}_2 \in \widetilde{U}_2$. Since π identifies points of $\widetilde{X}$ if and only if they are in the same $\widetilde{H}$-orbit, there is some $\varphi \in \widetilde{H}$ such that $\widetilde{q}_2 = \varphi(\widetilde{q}_1)$. Then φ maps $\widetilde{U}_1$ homeomorphically onto $\widetilde{U}_2$, so $\pi(\widetilde{U}_2) = \pi \circ \varphi(\widetilde{U}_1) = \pi(\widetilde{U}_1)$. This shows that any such sets U'_1, U'_2 are either disjoint or equal. Since π is surjective, $p'^{-1}(U)$ is equal to the disjoint union of the sets $\pi(\widetilde{U})$ as $\widetilde{U}$ ranges over the components of $p^{-1}(U)$.

  It remains only to show that for any such set $U' = \pi(\widetilde{U})$, $p' \colon U' \to U$ is a homeomorphism. The following diagram commutes:

  $$\begin{tikzcd} \widetilde{U} \arrow[d, "p"'] \arrow[dr, "\pi"] & \\ U. & U' \arrow[l, "p'"] \end{tikzcd} \tag{12.8}$$

  Since $p = p' \circ \pi$ is injective on $\widetilde{U}$, so is π; and $\pi \colon \widetilde{U} \to U'$ is surjective by definition. Because π is an open map, it follows that $\pi \colon \widetilde{U} \to U'$ is a homeomorphism. Since p and π are homeomorphisms in (12.8), so is p'.

- **Page 287, line 10:** The sentence "Thus (i) corresponds to the rank 1 case" should read "Thus (ii) corresponds to the rank 1 case."

- **Page 289, Problem 12-5:** Replace the statement of the problem by "Find a group Γ acting freely and properly on the plane such that $\mathbb{R}^2/\Gamma$ is homeomorphic to the Klein bottle."

- **Page 290, Problem 12-9:** Replace the second sentence by "For any element $\widetilde{e}$ in the fiber over the identity element of G, show that $\widetilde{G}$ has a unique group structure such that $\widetilde{e}$ is the identity, $\widetilde{G}$ is a topological group, and the covering map $p \colon \widetilde{G} \to G$ is a homomorphism with discrete kernel."

- **Page 301, just above the third displayed equation:** In the last sentence of the paragraph, replace $G_{i,p} \colon \Delta_p \to \Delta_p \times I$ by $G_{i,p} \colon \Delta_{p+1} \to \Delta_p \times I$.

- **Page 316, first paragraph:** Change the fourth sentence to: "For $p > 0$, if $\alpha \colon \Delta_p \to \mathbb{R}^n$ is an affine p-simplex, set

  $$s\alpha = \alpha(b_p) * s\partial\alpha$$

  (where b_p is the barycenter of Δ_p), and extend linearly to affine chains."

- **Page 319, statement of Lemma 13.21:** H^{n-1} should be H_{n-1}.

- **Page 320, first paragraph:** In the last two lines, H^{n-1} should be H_{n-1} (twice).

- **Page 325, second to last displayed equation:** Change $H_p(\mathcal{K}'')$ to $H_p^{\Delta}(\mathcal{K}'')$.

- **Page 330, paragraph after Exercise 13.4:** Replace [Mun75] by [Mun84].

- **Page 332, line 1:** The first word on the page should be "subgroups" instead of "spaces."

- **Page 333, line 7:** Change "coboundary" to "cocycle."

- **Page 335, Problem 13-12:** Add the hypothesis that $U \cup V = X$.
- **Page 344, Exercise A.7(a):** Since this exercise requires the axiom of choice, it should be moved after exercise A.9.