

CORRECTIONS TO
Introduction to Topological Manifolds
(First edition)

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Changes or additions made in the past twelve months are dated.

- **Page 29, statement of Lemma 2.11:** The second sentence should be replaced by “*If the open subsets of X are exactly those sets that satisfy the basis criterion with respect to \mathcal{B} , then \mathcal{B} is a basis for the topology of X .*”
- **Page 29, paragraph before Exercise 2.15:** Instead of “the topologies of Exercise 2.1,” it should say “some of the topologies of Exercise 2.1.”
- **Page 30, last sentence of the proof of Lemma 2.12:** Replace U by $f^{-1}(U)$ (three times).
- **Page 30, first paragraph in the “Manifolds” section:** Delete the sentence “Let X be a topological space.”
- **Page 38, Problem 2-16(b):** Replace part (b) by “Show that for any space Y , a map $f: X \rightarrow Y$ is continuous if and only if $p_n \rightarrow p$ in X implies $f(p_n) \rightarrow f(p)$ in Y .”
- **Page 38, Problem 2-18:** This problem should be moved to Chapter 3, because $\text{Int } M$ and ∂M are to be interpreted as having the subspace topologies. Also, for this problem, you may use without proof the fact that $\text{Int } M$ and ∂M are disjoint.
- **Page 40, last line of Example 3.1:** Replace “subspace topology on B ” by “subspace topology on C .”
- **Page 51, proof of Proposition 3.13, third line:** $f_1(U_1), \dots, f_k(U_k)$ should be replaced by $f_1^{-1}(U_1), \dots, f_k^{-1}(U_k)$.
- **Page 51, proof of Proposition 3.14, last sentence:** Replace “the preceding lemma” by “the preceding proposition.”
- **Page 52, first paragraph after Exercise 3.8:** In the first sentence, replace the words “surjective and continuous” by “surjective.” Also, add the following sentence at the end of the paragraph: “It is immediate from the definition that every quotient map is continuous.”
- **Page 52, last paragraph:** Change the word “quotient” to “surjective” in the first sentence of the paragraph.
- **Page 53, line 1:** Change the word “quotient” to “surjective” at the top of the page.
- **Page 53, Lemma 3.17:** Add the following sentence at the end of the statement of the lemma: (*More precisely, if $U \subset X$ is a saturated open or closed set, then $\pi|_U: U \rightarrow \pi(U)$ is a quotient map.*)
- **Page 82, line 3 from bottom:** Delete “ $= \overline{U} \cap Z$ ” from the sentence beginning “Since $\overline{U} \cap \overline{Z} \dots$ ”
- **Page 83, Example 4.30(a):** In the first sentence, change “closed” to “open” and change $\overline{B}_\varepsilon(x)$ to $B_\varepsilon(x)$.
- **Page 85, statement of Corollary 4.34:** “*countable collection*” should read “*countable union*.”

- **Page 99, Lemma 5.4:** Replace part (d) by
 - (d) For any topological space Y , a map $F: |\mathcal{K}| \rightarrow Y$ is continuous if and only if its restriction to $|\sigma|$ is continuous for each $\sigma \in \mathcal{K}$.
- **Page 103, Proposition 5.11:** In the statement of the proposition, change “simplicial complex” to “1-dimensional simplicial complex.”
- **Page 106, line 3 from bottom:** Replace “even” by “odd.”
- **Page 111, Figure 5.12:** In $S(SK)$, the points inside the small triangles should be at the intersections of the three medians.
- **Page 114, Problem 5-2:** Replace the statement of the problem by: “Let \mathcal{K} be an abstract simplicial complex. For each vertex v of \mathcal{K} , let $\text{St } v$ (the *open star* of v) be the union of the open simplices $\text{Int } |\sigma|$ as σ ranges over all simplices that have v as a vertex; and define a function $t_v: |\mathcal{K}| \rightarrow \mathbb{R}$ by letting $t_v(x)$ be the coefficient of v in the formal linear combination representing x .
 - (a) Show that each function t_v is continuous.
 - (b) Show that $\text{St } v$ is a neighborhood of v , and the collection of open stars of all the vertices is an open cover of $|\mathcal{K}|$.”
- **Page 114, Problem 5-3:** Delete the phrase “and locally path connected.”
- **Page 120, Statement of Proposition 6.2(b):** Replace $x \in \partial \mathbb{B}^2$ by $(x, y) \in \partial \mathbb{B}^2$.
- **Page 126, Proposition 6.6:** Add the hypothesis that $n \geq 2$.
- **Page 131, Part 1 of the definition of the geometric realization:** After “sides of length 1,” insert “equal angles.”
- **Page 136, line 8 from bottom:** Change the surface presentation in that line to $\langle S_1, S_2, a, b, c \mid W_1 c^{-1} b^{-1} a^{-1}, abcW_2 \rangle$.
- **Page 139, proof of the classification theorem:** Replace the first sentence of the proof with “Let M be the compact surface determined by the given presentation.”
- **Page 140, line 14:** Change “Step 3” to “Step 2.”
- **Page 149, Example 7.3:** The first line should read “Define maps $f, g: \mathbb{R} \rightarrow \mathbb{R}^2$ by . . .”
- (9/16/11) **Page 155, line 3:** Change $\Phi_g(f)$ to $\Phi_g[f]$.
- **Page 156, Figure 7.7:** The labels $I \times I$, F , and X should all be in math italics.
- **Page 156, Exercise 7.2:** Change the first sentence to “Let X be a path connected topological space.”
- **Page 159, second line from bottom:** “induced homeomorphism” should read “induced homomorphism.”
- **Page 160, Proposition 7.18:** In the statement and proof of the proposition, change $(\iota_A)^*$ to $(\iota_A)_*$ three times (the asterisk should be a subscript).
- **Page 174, proof of Lemma 7.35:** In the second-to-last line of the proof, change “Theorem 3.10” to “Theorem 3.11.”
- **Page 176, Problem 7-5:** Change “compact surface” to “connected compact surface.”

- **Page 188, proof of Theorem 8.7:** Replace the third sentence of the proof by “If $f: I \rightarrow \mathbb{S}^n$ is any loop based at a point in $U \cap V$, by the Lebesgue number lemma there is an integer m such that on each subinterval $[k/m, (k+1)/m]$, f takes its values either in U or in V . If $f(k/m) = N$ for some k , then the two subintervals $[(k-1)/m, k/m]$ and $[k/m, (k+1)/m]$ must be both mapped into V . Thus, letting $0 = a_0 < \dots < a_l = 1$ be the points of the form k/m for which $f(a_i) \neq N$, we obtain a sequence of curve segments $f|_{[a_{i-1}, a_i]}$ whose images lie either in U or in V , and for which $f(a_i) \neq N$.” Also, in the last line of the proof, replace “ f is homotopic to a path” by “ f is path homotopic to a loop.”
- **Page 189, proof of Proposition 8.9:** In the last sentence of the proof, change the domain of H to $I \times I$, and change the definition of H to

$$H(s, t) = (H_1(s, t), \dots, H_n(s, t)).$$

- **Page 191, Problem 8-7:** In the third line of the problem, change $\varphi(\gamma)$ to $\varphi_*(\gamma)$.
 - **Page 192, line 4:** Change the definition of φ to $\varphi(x) = (x - f(x))/|x - f(x)|$.
 - **Page 199, second-to-last paragraph:** In the second sentence, after “a product of elements of S ,” insert “or their inverses.”
 - **Page 208, Problem 9-4(b):** Change the first phrase to “Show that $\text{Ker } f_1 * f_2$ is equal to the normal closure of $\text{Im } j_1 * j_2, \dots$ ” Add the following hint: “[Hint: Let N denote the normal closure of $\text{Im } j_1 * j_2$, so it suffices to show that $f_1 * f_2$ descends to an isomorphism from $(G_1 * G_2)/N$ to $H_1 * H_2$. Construct an inverse by showing that each composite map $G_j \hookrightarrow G_1 * G_2 \rightarrow (G_1 * G_2)/N$ passes to the quotient yielding a map $H_j \rightarrow (G_1 * G_2)/N$, and then invoking the characteristic property of the free product.]”
 - **Page 213, proof of Proposition 10.5:** In the second sentence of the proof, change $\{q\}$ to $\{*\}$.
 - **Page 218, Figure 10.4:** In the upper diagram, one of the arrows labeled a_i should be reversed.
 - **Page 227, line 8:** Replace $\overline{R} * \overline{S}$ by $\overline{R * S}$.
 - **Page 233, last line:** Change the last sentence to “This brings us to the next-to-last major subject in the book: . . .”
 - **Page 238, proof of Proposition 11.10, second line:** Change “ p maps . . .” to “ f maps . . .”
 - **Page 248, Example 11.26:** Change $\mathcal{C}_\pi(\mathbb{P}^n)$ to $\mathcal{C}_\pi(\mathbb{S}^n)$.
 - **Page 249, line 5:** Change the formula to “ $p(\varphi(\tilde{q})) = p(\tilde{q}) = q$ ” (not p).
 - **Page 253, Problem 11-9:** Change “path connected” to “locally path connected.”
 - **Page 265, Step 4:** In the second line of Step 4, replace “as in Step 3” by “as in Step 2.”
 - **Page 268, proof of Theorem 12.11:** The first and last paragraphs of this proof can be simplified considerably by using the result of Problem 3-15.
 - **Page 272, first paragraph:** The last sentence should read “It can be identified with a quotient of the group of matrices of the form $\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$ (identifying two matrices if they differ by a scalar multiple), and so is a topological group acting continuously on \mathbb{B}^2 .”
- (6/21/11) **Page 284, just below equation (12.8):** Replace the entire sentence starting with “If $\pi(\tilde{U}) \neq U$ ” (including the last displayed equation on the page) with the following: “Since $\pi \circ \varphi = \pi$ for $\varphi \in \hat{H}$, it follows that $\pi(\varphi(\tilde{U})) = \pi(\tilde{U})$, so the images of all the sets $\varphi(\tilde{U})$ for φ in \hat{H} are the same. Therefore, since π is surjective, so is its restriction to \tilde{U} .”

- **Page 287, line 10:** The sentence “Thus (i) corresponds to the rank 1 case” should read “Thus (ii) corresponds to the rank 1 case.”
- **Page 289, Problem 12-5:** Replace the statement of the problem by “Find a group Γ acting freely and properly on the plane such that \mathbb{R}^2/Γ is homeomorphic to the Klein bottle.”
- **Page 290, Problem 12-9:** Replace the second sentence by “For any element \tilde{e} in the fiber over the identity element of G , show that \tilde{G} has a unique group structure such that \tilde{e} is the identity, \tilde{G} is a topological group, and the covering map $p: \tilde{G} \rightarrow G$ is a homomorphism with discrete kernel.”
- **Page 301, just above the third displayed equation:** In the last sentence of the paragraph, replace $G_{i,p}: \Delta_p \rightarrow \Delta_p \times I$ by $G_{i,p}: \Delta_{p+1} \rightarrow \Delta_p \times I$.
- **Page 316, first paragraph:** Change the fourth sentence to: “For $p > 0$, if $\alpha: \Delta_p \rightarrow \mathbb{R}^n$ is an affine p -simplex, set

$$s\alpha = \alpha(b_p) * s\partial\alpha$$
 (where b_p is the barycenter of Δ_p), and extend linearly to affine chains.”
- **Page 319, statement of Lemma 13.21:** H^{n-1} should be H_{n-1} .
- **Page 320, first paragraph:** In the last two lines, H^{n-1} should be H_{n-1} (twice).
- **Page 325, second to last displayed equation:** Change $H_p(\mathcal{K}'')$ to $H_p^\Delta(\mathcal{K}'')$.
- **Page 330, paragraph after Exercise 13.4:** Replace [Mun75] by [Mun84].
- **Page 332, line 1:** The first word on the page should be “subgroups” instead of “spaces.”
- **Page 333, line 7:** Change “coboundary” to “cocycle.”
- **Page 335, Problem 13-12:** Add the hypothesis that $U \cup V = X$.
- **Page 344, Exercise A.7(a):** Since this exercise requires the axiom of choice, it should be moved after exercise A.9.