Corrections to
Introduction to Smooth Manifolds, First Edition ©2006
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Changes or additions made in the past twelve months are dated.

- **Page 6, line 5:** Replace $\mathbb{R}^n$ by $\mathbb{R}^{n+1}$.
- **Page 6, lines 6 and 3 from the bottom:** Replace $U_i^+ \cap \mathbb{S}^n$ by U_i^+, and replace $U_i^- \cap \mathbb{S}^n$ by U_i^-.
- **Page 7, lines 1 and 2:** Replace $U_i^\pm \cap \mathbb{S}^n$ by $U_i^\pm$ (twice).
- **Page 9, last line of first paragraph:** Insert "is" before "the same."
- **Page 10, last paragraph, fifth line:** Change $B \subset \mathcal{B}$ to $B \in \mathcal{B}$.
- **Page 10, last line:** Replace $\widetilde{f}_n$ by $\widetilde{f}_k$.
- **Page 17, Example 1.14:** In the sixth line of this example, replace $\mathrm{Id}_{\mathbb{R}^n}$ by $\mathrm{Id}_{\mathbb{R}}$.
- **Page 18, second line below the heading:** After "throughout the book," insert "(except in the Appendix, which is designed to be read before the rest of the book)."
- **Page 19, line 6 from the bottom:** Replace A_U by $\mathcal{A}_U$.
- **Page 21, second-to-last paragraph:** Change $\varphi_\alpha \circ \varphi_\beta^{-1}(W)$ to $(\varphi_\beta \circ \varphi_\alpha^{-1})^{-1}(W)$ twice: once in the text, and once in the displayed equation.
- **Page 22, Figure 1.9:** Replace $\varphi(U_\alpha)$ by $\varphi_\alpha(U_\alpha)$, $\varphi(U_\beta)$ by $\varphi_\beta(U_\beta)$, and $\varphi_\alpha \circ \varphi_\beta^{-1}(W)$ by $(\varphi_\beta \circ \varphi_\alpha^{-1})^{-1}(W)$.
- **Page 25, line 7 from bottom:** After "$V \subset \mathbb{R}^n$," insert "containing x."
- **Page 31, line 2 from bottom:** Change "smooth maps" to "smooth functions."
- **Page 33, proof of Lemma 2.2, fourth line:** Replace $\varphi(V)$ by $\psi(V)$.
- **Page 38, Example 2.7(b), first line:** Insert "invertible" before "complex."
- **Page 41, line 2:** After the period, insert the sentence "It is called a *universal covering manifold* if $\widetilde{M}$ is simply connected."
- **Page 50, line 7 from the bottom:** Replace f by $f^{(k)}$.
- **Page 54, first full paragraph:** I think the argument is clearer if that paragraph is replaced by the following: "Each set W_k^i is contained in some V_j, which must satisfy $V_j \cap \overline{V}_k \neq \varnothing$ and consequently $V_j \cap V_k \neq \varnothing$. Therefore, if $W_k^i \cap W_{k'}^{i'} \neq \varnothing$, there are indices j and j' such that $V_j \cap V_{j'}$, $V_j \cap V_k$, and $V_{j'} \cap V_{k'}$ are all nonempty. By the result of Problem 12-14, for each fixed k, there can only be finitely many such indices k', and thus only finitely many sets $W_{k'}^{i'}$ can intersect W_k^i. Thus the cover $\{W_k^i\}$ is locally finite."
- **Page 56, line just above the second displayed equation:** Change "for each p" to "for each p such that $\psi_p(x) \neq 0$," followed by a comma.
- **Page 57, Problem 2-1:** In the introductory paragraph, change "representation" to "representations." In part (c), change $F(z, w)$ to $F(w, z)$.

- **Page 62, Figure 3.1:** Change V_a to v_a (lowercase v).
- **Page 62, last paragraph, line 3:** change "Euclidean tangent vector" to "geometric tangent vector."
- **Page 67, line 4 from the bottom:** Delete redundant "the."
- **Page 74, line above the first displayed equation:** Insert "on some neighborhood of a" after "$\widetilde{f} \circ \iota = f$" and before the comma.
- **Page 74, line below the first displayed equation:** Replace $\mathbb{T}_a\mathbb{R}^n$ by $T_a\mathbb{R}^n$.
- **Page 89, Proof of Proposition 4.10:** It's much easier to prove smoothness of Z by noting that it's equal to the composition of smooth maps $F_* \circ Y \circ F^{-1}$.
- **Page 89, line 9:** Change "preceding lemma" to "preceding proposition."
- **Page 90, line 2:** WVf should be VWf.
- **Page 91, lines 1 and 2:** Change "its values are determined locally" to "its action on a function is determined locally."
- **Page 93, two lines above Lemma 4.18:** Change $\mathcal{T}(M)$ to $\mathcal{T}(G)$.
- **Page 95, line 4 from the bottom, and page 96, line 1:** Change $(-\varepsilon, \varepsilon)$ to $(-\delta, \delta)$ (to avoid conflict with the evaluation map ε).
- **Page 96, proof of Corollary 4.21:** The following rephrasing might be clearer: "Let Y be a left-invariant rough vector field on a Lie group G, and let $V = Y_e$. The fact that Y is left-invariant implies that $Y = \widetilde{V}$, which is smooth."
- **Page 100, Proposition 4.26:** In the statement of part (a) and in the first line of the proof, replace $(\mathrm{Id}_G)*$ by $(\mathrm{Id}_G)_*$ (the asterisk should be set as a subscript).
- **Page 100, proof of Proposition 4.26, next to last line:** Should read "$F_* \circ \left(F^{-1}\right)_* = \left(F \circ F^{-1}\right)_* = \mathrm{Id}_{\mathrm{Lie}(H)}$ and $\left(F^{-1}\right)_* \circ F_* = \mathrm{Id}_{\mathrm{Lie}(G)}$."
- **Page 102, Problem 4-13:** The algebra A does not need to be assumed to be associative.
- **Page 109, line 4 below the section heading:** Replace "each $p \in U$" by "each $p \in M$."
- **Page 114, proof of Lemma 5.14, third line from bottom:** Change $\varphi \times \mathrm{Id}_U$ to $\varphi \times \mathrm{Id}_{\mathbb{R}^n}$.
- **Page 115, just above the section heading:** Delete the sentence beginning "A remarkable theorem ...." [*The theorem actually proved by Wolf requires an extra consistency hypothesis in addition to parallelizability, so it does not apply to arbitrary compact, simply connected parallelizable manifolds. Similar misstatements of Wolf's theorem are common in the literature, so be careful whenever you see this theorem quoted. My apologies for perpetuating the misunderstanding.*]
- **Page 117, last displayed equation:** Replace $\mathcal{A}_i^j(q)$ by $A_i^j(q)$.
- **Page 117, last line:** Replace "trivializations of E" by "trivializations of E and E'."
- **Page 118, 6 lines above the section heading:** Change C^∞ to $C^\infty(M)$.
- **Page 122, Problem 5-11:** The stated conditions are not sufficient to guarantee the uniqueness of the smooth structure on T. Instead, show that T can be identified with a subset of $\mathrm{G}_k(V) \times V$; show that T is a smooth subbundle of this trivial bundle, and use this to define the smooth structure on it.
- **Page 127, first line:** Replace V^* by V^{**}.

- **Page 129, statement of Proposition 6.5, first line:** Change "*manifold*" to "*n-manifold.*"
- **Page 129, two lines above the last displayed equation:** Replace $\pi^{-1}(U \cap V)$ by $\pi^{-1}(U \cap \widetilde{U})$.
- **Page 129, line 3 from the bottom:** Replace "Proposition 5.3" by "Lemma 5.14."
- **Page 131, line 3:** Replace (ε_i) by (ε^i).
- **Page 131, line 5:** Replace "vector fields" by "covector fields."
- **Page 135, Figure 6.2:** The axis label on the right should be $x^2, \dots, x^n$.
- **Page 135, first line of text:** The domain of $f \circ \gamma$ should be J, not $\mathbb{R}$.
- **Page 136, sentence just above equation (6.12):** Replace that sentence by "Given a smooth map $G\colon M \longrightarrow N$ and a covector field ω on N, define a rough covector field $G^*\omega$ on M by $\dots$." [Remark: Pullbacks of continuous covector fields make perfectly good sense; because many of the results on integration theory in Chapter 14 are stated without the hypothesis of smoothness, the pullback operation needs to be defined without assuming smoothness.]
- **Page 136, next to last line:** After "$G^*\omega$ is smooth," insert "when ω is."
- **Page 137, statement of Lemma 6.12:** Change the first sentence to "*Let $G\colon M \longrightarrow N$ be a smooth map, and suppose ω is a covector field on N and $f\colon N \longrightarrow \mathbb{R}$ is continuous.*" Modify equation (6.13) as follows:
$$G^* df = d(f \circ G) \qquad \text{if } f \text{ is smooth}; \tag{6.13}$$
- **Page 137, statement of Proposition 6.13:** Replace the statement by "*Suppose $G\colon M \longrightarrow N$ is smooth, and let ω be a covector field on N. Then $G^*\omega$ is a (continuous) covector field on M. If ω is smooth, then so is $G^*\omega$.*"
- **Page 137, proof of Proposition 6.13:** In the third line, replace "smooth functions" by "continuous functions." Replace the last line by "This expression is continuous, and is smooth if ω is smooth."
- **Page 139, third line after Exercise 6.8:** Change "if it is has" to "if it has."
- **Page 142, line 1:** Change "smooth curve segments" to "piecewise smooth curve segments."
- **Page 143, eighth line after the section heading:** Change "Lemma 6.10" to "Proposition 6.10."
- **Page 151, Problem 6-2:** Replace the problem statement by the following:
  (a) If $F\colon M \longrightarrow N$ is a diffeomorphism, show that $F^*\colon T^*N \longrightarrow T^*M$ is a smooth bundle map.
  (b) Show that the assignment $M \mapsto T^*M$, $F \mapsto F^*$ defines a contravariant functor from SM_1 to VB, where SM_1 is the subcategory of SM whose objects are smooth manifolds, but whose morphisms are only diffeomorphisms.
- **Page 152, Problem 6-8(b):** Add the following hint: "[Hint: Use Proposition 5.16, noting that $C^\infty(M)$ can be naturally identified with the space of smooth sections of the trivial line bundle $M \times \mathbb{R}$.]"
- **Page 153, Problem 6-13:** Change "compact manifold" to "compact positive-dimensional manifold."
- **Page 154, Problem 6-14:** Throughout the problem, change the index i to j, to avoid confusion with $i = \sqrt{-1}$ in $e^{2\pi it}$ and $e^{2\pi i x^j}$.
- **Page 158, first displayed equation:** There is a missing factor of c in an exponent in the second member of the equation. The entire line should read
$$\left|e^{2\pi ick} - 1\right| = \left|e^{-2\pi icn_2}\left(e^{2\pi icn_1} - e^{2\pi icn_2}\right)\right| = \left|e^{2\pi icn_1} - e^{2\pi icn_2}\right| < \varepsilon.$$

- **Page 160, two lines above (7.1):** After "$|DH(x)| \le \frac{1}{2}$," insert "and $DF(x) = \mathrm{Id} - DH(x)$ is invertible."

- **Page 163, just before line 3 from the bottom:** Insert the following sentence: "By shrinking U_0 and $\widetilde{U}_0$ if necessary, we may assume that $\widetilde{U}_0$ is an open rectangle."

- **Page 164, line 7:** The range of $\widetilde{R}$ should be $\mathbb{R}^{n-k}$, not $\mathbb{R}^k$.

- **Page 164, just above equation (7.9):** Just before "Thus," insert the following parenthetical remark: "(This is one reason we need $\widetilde{U}_0$ to be a rectangle.)"

- **Page 164, just above the last displayed equation:** Before "It follows," insert the following sentence: "The fact that $\widetilde{U}_0$ is a rectangle guarantees that $F\circ\varphi^{-1}(\widetilde{U}_0) \subset V_0$, and therefore $F(U_0) \subset V_0$."

- **Page 171, Problem 7-1:** At the end of the problem statement, add this sentence: "Specifically, show that X passes to the quotient to define a smooth map $\widetilde{X}\colon \mathbb{T}^2 \to \mathbb{R}^3$, and then show that $\widetilde{X}$ is a smooth embedding whose image is the given surface of revolution."

- **Page 172, Problem 7-10, line 7:** Change the sentence beginning "Show that" to read "Show that $M_1\#M_2$ is a topological n-manifold, which is connected if $n \ge 2$, and ...."

- **Page 172, Problem 7-10, line 8:** Delete "unique."

- **Page 175, line below the first three displayed equations:** Replace the phrase "both φ and π are open maps" by "$\varphi(V)$ is the intersection of $\varphi(U)$ with an affine subspace $A \subset \mathbb{R}^n$ and is therefore open in A, and $\pi|_A$ is a homeomorphism."

- **Page 177, proof of Theorem 8.3:** In the second line of the proof, replace U by V (twice).

- **Page 186, line 10 from the bottom:** Change N to S (twice).

- **Page 189, third line after the subheading:** Change "a immersion" to "an immersion."

- **Page 192, proof of Proposition 8.27, second paragraph:** In the first three lines of that paragraph, change p to p_0.

- **Page 192, fourth line from the bottom:** The symbol S is a bad choice for the slice $V \cap U$, because it is already in use for the submanifold $S \subset M$. Change every occurrence of S in the last four lines of page 192 and the first three lines of page 193 to another symbol, say W. (There are six S's that need to be changed. But note that the S on line 4 of page 193 should not be changed.)

- **Page 192, last line:** Change "by the extension lemma, Lemma 2.27" to "by the result of Problem 8-11(a)."

- **Page 193, proof of Corollary 8.28:** In the first line of the proof, change "preceding lemma" to "preceding proposition." In the last line of the proof, change "and is therefore tangent to S" to "which is therefore tangent to S."

- **Page 196, Example 8.34, first line:** Add "$\mathrm{SL}(n,\mathbb{R})$" after the words "The set."

- **Page 198, equation (8.3) and last line:** Change $T_{I_n}\,\mathrm{GL}(2n,\mathbb{R})$ to $T_{I_{2n}}\,\mathrm{GL}(2n,\mathbb{R})$ twice: once in (8.3), and once in the last line on the page.

- **Page 200, proof of Lemma 8.41, 5th and 6th lines:** Change D_p to D_q and $p \in U$ to $q \in U$.

- **Page 201, line 3:** Change the definition of D_p to $D_p = \mathrm{span}(E_1|_p, \dots, E_k|_p)$.

- **Page 208, Example 9.1(b), next to last line:** Change "by some linear transformation" to "by some invertible linear transformation."
- **Page 209, Example 9.1(e), line 5:** Replace "any $v \in \mathbb{S}^n$" by "any $v \in \mathbb{S}^{n-1}$."
- **Page 217, line 4:** Replace $\lim_i(g_i \cdot q_i)$ by $\lim_i(g_i \cdot p_i)$.
- **Page 217, line 5:** Replace $\overline{U} \times \overline{V}$ by $\overline{V} \times \overline{U}$.
- **Page 226, last line before Theorem 9.19:** The period should go inside the right parenthesis.
- **Page 229, Exercise 9.6:** For $\mathrm{SU}(n)$, add the hypothesis that $n > 1$.
- **Pages 229–230, proof of Theorem 9.22:** There is a serious gap in the proof that H acts properly on G. Although the sequence $\{h_i\}$ constructed at the top of page 230 converges in G to a point in H, it does not follow that it converges in the topology of H, since we are not assuming that H is embedded. What is needed is the following converse to Proposition 8.30. (This converse is an immediate consequence of Theorem 20.10, the closed subgroup theorem, but that is not available in Chapter 9.)

  Proposition. *If G is a Lie group and $H \subset G$ is a closed Lie subgroup, then H is embedded.*

  Sketch of Proof. The result is trivial if $\dim H = \dim G$, so we may assume that H has positive codimension in G. It suffices to show that there is some point $h_1 \in H$ and a neighborhood of h_1 in G in which H is embedded, for then composition with left translation yields a slice chart centered at any other point of H. By Lemma 8.18, there exist a neighborhood V of e in H and a slice chart (U, φ) for V in G centered at e. We may assume that $\varphi(U) = B_1 \times B_2$, where B_1 and B_2 are Euclidean balls and $\varphi(V) = B_1 \times \{0\}$. The set $S = \varphi^{-1}(\{0\} \times B_2)$ is an embedded submanifold of G. By the inverse function theorem, the map $\psi\colon V \times S \to G$ obtained by restricting group multiplication, $\psi(v, s) = vs$, is a diffeomorphism from a product open set $V_0 \times S_0 \subset V \times S$ to a neighborhood U_0 of e.

  Let $K = S_0 \cap H$. Careful analysis of ψ shows that $\psi(V_0 \times K) = H \cap U_0$, and that K is discrete in the topology of H and thus countable. Because H is closed in G, it follows that K is closed in S_0. By an easy application of the Baire category theorem, it follows that a closed, countable subset of a manifold must have an isolated point, so there is a point $h_1 \in K$ that is isolated in S_0. This means there is a neighborhood S_1 of h_1 in S_0 such that $S_1 \cap H = \{h_1\}$, so $U_1 = \psi(V_0 \times S_1)$ is a neighborhood of h_1 in G with the property that $U_1 \cap H$ is a slice.
- **Page 231, Example 9.25:** At the end of the last sentence of part (a), add "when $n \geq 2$."
- **Page 233, lines 1 & 2:** Replace "quotient manifold theorem" by "homogeneous space construction theorem."
- **Page 233, lines 9 & 10:** Change the last sentence of the proof to "By Proposition 7.18, $\widetilde{F}$ is smooth, and by the equivariant rank theorem together with Theorem 7.15, it is a diffeomorphism."
- **Page 233, line 4 from the bottom:** Replace "inverse function theorem" by "constant-rank level set theorem."
- **Page 237, Figure 9.7:** The label $\mathrm{O}(n)$ should be replaced by $\mathrm{SO}(n)$.
- **Page 240, Problem 9-26:** Change $\mathrm{SU}(n) \times \mathbb{R}^{n^2}$ to $\mathrm{SU}(n) \times \mathbb{R}^{n^2-1}$.
- **Page 240, last line:** Change $C_\pi(\widetilde{M})$ to $\mathcal{C}_\pi(\widetilde{M})$.
- **Page 250, line 7 from the bottom:** The second occurrence of F_{k-1} should be F_k.
- **Page 254, second line from the bottom:** Change $\varphi(NM \cap U)$ to $\widetilde{\varphi}\big(NM \cap \pi^{-1}(U)\big)$.

- **Page 257, proof of Proposition 10.20, first line:** Change "containing M" to "containing M_0."
- **Page 257, proof of Theorem 10.21, lines 3 and 4:** Change "Lemma 10.20" to "Proposition 10.20."
- **Page 258, proof of Proposition 10.22, just above the displayed equation:** The range of $\overline{H}$ should be N, not M.
- **Page 259, Problem 10-4:** Change $\mathbb{R}^m$ to $\mathbb{R}^n$.
- **Page 263, second line from bottom:** Change "arbitrary" to "appropriate."
- **Page 270, line 2:** The end of the second line is missing an F and a colon. It should end "called the pullback by F:"
- **Page 270, Proposition 11.8(e):** Add the hypothesis that F is a diffeomorphism.
- **Page 270, Exercise 11.8:** Change "Proposition 11.9" to "Proposition 11.8."
- **Page 270, paragraph after Exercise 11.8:** In the first sentence of the paragraph, replace "the category of smooth manifolds and smooth maps" by "the subcategory SM_1 of SM consisting of smooth manifolds and diffeomorphisms."
- **Page 270, middle of the page, paragraph beginning "Just as in":** In the second line of the paragraph, delete "smooth" before "tensor fields." Change the third line to "For any covariant k-tensor field σ on N, we define a rough k-tensor field . . . ."
- **Page 270, statement of Proposition 11.9:** Replace the phrase "$\sigma \in \mathcal{T}^k(N)$, $\tau \in \mathcal{T}^l(N)$, *and* $f \in C^\infty(N)$" by "σ *and* τ *are covariant tensor fields on* N*, and* $f\colon N \to \mathbb{R}$ *is continuous.*" Replace the statement of part (c) with "$F^*\sigma$ *is a* (*continuous*) *tensor field, and is smooth if* σ *is smooth.*"
- **Page 271, paragraph after Exercise 11.9:** In the first two lines of that paragraph, change the first instance of "smooth" to "continuous," and delete the second and third occurrences of "smooth."
- **Page 271, statement of Corollary 11.10:** Change "*let* $\sigma \in \mathcal{T}^k(N)$" to "*let* σ *be a covariant* k*-tensor field on* N*.*"
- **Page 277, proof of Proposition 11.17, third line:** Change "Proposition 10.17" to "Lemma 10.17."
- **Page 279, last paragraph:** Replace the first three sentences in this paragraph with the following: "Conversely, suppose that W is open in the metric topology, and let $p \in W$. Let $\overline{V}$ be a smooth coordinate ball of radius r around p, let $\overline{g}$ be the Euclidean metric on $\overline{V}$ determined by the given coordinates, and let c, C be positive constants such that (11.6) is satisfied for $X \in T_qM$, $q \in \overline{V}$. Let $\varepsilon < r$ be a positive number small enough that the metric ball around p of radius $C\varepsilon$ is contained in W, and let V_ε be the set of points in $\overline{V}$ whose Euclidean distance from p is less than ε."
- **Page 281, line 3 from the bottom:** Change "Proposition 10.17" to "Lemma 10.17."
- **Page 282, Exercise 11.16:** Add the following sentence at the end of the exercise: "(The same is true when S is merely immersed; the proof is a bit more complicated but straightforward.)"
- **Page 287, Problem 11.11(a):** Change "isomorphic vector bundles" to "isomorphic vector bundles over M."
- **Page 293, last displayed equation:** All of the occurrences of n in this equation should be changed to k (5 times). Also, the following n's should be changed to k's: Last two lines of page 293 (2 times); first line of page 294 (1 time); displayed equation in the proof of Lemma 12.1 (7 times).

- **Page 298, line 5:** Change the first "multi-index" to "index."
- **Page 303, third displayed equation:** In this equation and in the line above it, change ω^I to ω_I.
- **Page 303, paragraph before Lemma 12.10:** In the first two lines of the paragraph, replace "smooth differential form" by "differential form" (twice). In the third line, delete "smooth."
- **Page 304, statement of Proposition 12.12, third line:** Change "*smooth*" to "*continuous*."
- **Page 305, proof of Corollary 12.13:** Change G to F.
- **Page 307, last line:** The sentence beginning "Properties (a)–(c) ..." should be moved to the end of the next paragraph, after "This completes the proof of the existence and uniqueness of d in this special case." (Until d is proved to be unique, we don't know that it is given by (12.15) in *every* smooth chart.)
- **Page 308, last displayed equation:** Add one extra equality, so that the entire equation reads

$$0 = \widetilde{d}(\varphi\eta)_p = \widetilde{d}\varphi_p \wedge \eta_p + \varphi(p)\widetilde{d}\eta_p = d\varphi_p \wedge \eta_p + \varphi(p)\widetilde{d}\eta_p = \widetilde{d}\eta_p.$$

- **Page 308, line 3 from the bottom:** Replace $d\eta$ by $\widetilde{d}\eta$.
- **Page 311, displayed equation above Exercise 12.6:** The right-hand side should be summed only over $j < k$:

$$d\varepsilon^i = -\sum_{j<k} c^i_{jk}\, \varepsilon^j \wedge \varepsilon^k.$$

- **Page 313, just below the second displayed equation:** Replace "The only terms ..." by "If J has a repeated index, then both sides are zero. Otherwise, the only terms ... ."
- **Page 315, Exercise 12.8(d):** The last part of the sentence should read "and $\dim S = \frac{1}{2}\dim V$."
- **Page 317, line above the first displayed equation:** Replace $T_{(q,\varphi)}(T^*Q)$ by $T^*_{(q,\varphi)}(T^*Q)$.
- **Page 318, Figure 12.4:** The label ${}_qQ$ should be T_qQ.
- **Page 321, First line after the displayed commutative diagram:** Change the beginning of this sentence to "Define a wedge product on $\bigoplus_k A^k(V^*)$ by ... ."
- **Page 321, Line 5 after the displayed commutative diagram:** Change $\Lambda^k(V)$ to $\Lambda^*(V)$.
- **Page 322, Problem 12-14:** The matrix J should be defined as

$$J = \begin{pmatrix} j & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & j \end{pmatrix},$$

with copies of the 2×2 block $j = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ along the main diagonal, and zeros elsewhere.

- **Page 323, Problem 12-17:** Add the hypothesis that the forms $\alpha^1, \dots, \alpha^k$ are smooth.
- **Page 337, proof of Proposition 13.12, line 3 from the bottom:** Change E_n to E_{n-1}.
- **Page 339, proof of Lemma 13.16, first line:** After "neighborhood of ∂M," insert "in M."
- **Page 340, proof of Proposition 13.17:** The proof of independence of N given here does not work in the case $n = 1$. Instead, in that case, just notice that any other outward-pointing vector field along ∂M is equal to N multiplied by a positive function, so the sign of $N \lrcorner \Omega$ at each point of ∂M is independent of the choice of N.

- **Page 342, line just below the second displayed equation:** Change "$\widetilde{X}(1,\theta,\rho) = X(\theta,\rho)$" to "$\widetilde{X}(1,\varphi,\theta) = X(\varphi,\theta)$."
- **Page 342, last line of Example 13.21:** Replace D by $(0,1] \times U$.
- **Page 347, Problem 13-7:** Insert "M" after "$2n$-manifold" and before the comma.
- **Page 347, Problem 13-9(a):** Add the following sentence at the end of the problem statement: "(You may use without proof the fact that the result of Exercise 11.16 holds also for immersed submanifolds.)"
- **Page 350, line 2 from bottom:** Insert "compact" before "domain of integration."
- **Page 352, proof of Corollary 14.3, second line:** Change "smooth maps" to "diffeomorphisms."
- **Page 356, line 7:** Change "any n-form" to "any compactly supported n-form."
- **Page 362, first displayed equation:** In the last term, replace b by a.
- **Page 362, Corollaries 14.11 and 14.12:** Add the hypothesis that M is oriented.
- **Page 362, Corollary 14.13:** Add the hypothesis that S is oriented, and change the conclusion to "…and S is not the boundary of a smooth, compact, oriented submanifold with boundary in M." [Add the word "oriented."]
- **Page 365, first full paragraph, first line:** Change $\psi(V)$ to $\psi(U \cap V)$ (twice).
- **Page 366, two lines above Proposition 14.18:** After the phrase "whose boundary has measure zero in ∂M," insert "and whose closure is compact."
- **Page 367, statement of Theorem 14.20:** Before "$(n-1)$-form," insert "smooth."
- **Page 378, statement of Proposition 14.30, third line:** Change "*smooth*" to "*continuous*."
- **Page 382, last line of the top displayed equation:** Replace dV_g by $dV_{\widetilde{g}}$.
- **Page 382, Problem 14-1:** Change the phrase "with the orientation determined by its product structure (see Exercise 13.4)" to "with the orientation determined by $d\theta^1 \wedge d\theta^2$, where θ^1 and θ^2 are angle coordinates on the first and second copies of $\mathbb{S}^1$, respectively."
- **Page 397, proof of Theorem 15.9:** In line 3 of the proof, replace "chain maps" by "cochain maps."
- **Page 408, Problem 15-6:** Add the hypothesis that N is connected.
- **Page 408, Problem 15-7:** Add the hypothesis that M_1 and M_2 are compact.

(6/5/18) **Page 408, Problem 15-11:** Replace the hint by the following: "[Hint: If ω is a closed 1-form on M, let $\widetilde{\omega}$ be the pullback of ω to the universal cover of M, let $\widetilde{f}$ be a potential function for ω, and define $f\colon M \to \mathbb{R}$ by letting $f(x)$ be the average value of $\widetilde{f}$ over the preimages of x.]"

- **Page 412, line 5:** Inside the parentheses, just before "Such a map," insert the following sentence: "An *affine map* $F\colon \mathbb{R}^p \to \mathbb{R}^m$ is a map of the form $F(x) = Tx + x_0$ for some linear map $T\colon \mathbb{R}^p \to \mathbb{R}^m$ and some fixed $x_0 \in \mathbb{R}^m$.
- **Page 413, line 11 from the bottom:** All three occurrences of F should be changed to $F_\#$, so the sentence begins "An easy computation shows that $F_\# \circ \partial = \partial \circ F_\#$, so $F_\#$ is …"
- **Page 413, line 8 from the bottom:** Change $(Id_M)_*$ to $(\mathrm{Id}_M)_*$ [Id should not be in italics].

- **Page 421, first paragraph:** After the first sentence, insert the following: "Because the argument is local from this point on, after shrinking U if necessary we may replace M with a coordinate neighborhood of $f(x)$ that is diffeomorphic to $\mathbb{R}^m$; thus we will henceforth identify M with $\mathbb{R}^m$."
- **Page 422, last paragraph, third line:** Replace $(e_1, 1)$ by $(e_i, 1)$.
- **Page 426, line 5:** Change $d\widetilde{b}$ to $\partial\widetilde{b}$.
- **Page 426, third displayed equation:** Replace ω by η in the third integral.
- **Page 430, line 10:** Replace "for m even" by "for $m \in \mathbb{Z}$."
- **Page 432, Problem 16-5(a):** Change ∂ to d, yielding $d \circ \iota_\# = \iota_\# \circ d$.
- **Page 438, three lines above equation (17.2):** Change "left action" to "continuous left action."
- **Page 439, proof of Proposition 17.3:** In the first line of the proof, replace "Lemma 4.2" by "Lemma 4.6."
- **Page 443, line 10 of the proof:** Insert "the" before "same ODE."
- **Page 449, equation (17.9), second line:** Add missing right parenthesis to $(f(\theta_t(0, u_0)))$, and change the partial derivative to an ordinary derivative (in the second line only), so it reads

$$= \left.\frac{d}{dt}\right|_{t=t_0} (f(\theta_t(0, u_0))) = V_{\psi(t_0,u_0)} f,$$

- **Pages 452–453, statement of Lemma 17.16:** Replace "*nonnegative constants A and B*" by "*constants $A > 0$ and $B \geq 0$.*"
- **Page 453, line in the middle of the page beginning "It is easy":** Change $g(t) > 0$ to $g(t) \geq 0$.
- **Pages 456–459, Theorem 17.19:** There are several errors in the proof of this theorem. First, the inequality on the first line of page 457 is incorrect, because $\theta(s, \widetilde{x})$ might not lie in $\overline{U}_1$ for all s between t and $\widetilde{t}$, so we cannot conclude that $|V(\theta(s, \widetilde{x}))| \leq M$. Second, (17.23) and (17.24) are only stated for $y \in \overline{U}_1$, but they are used in the middle of page 458 with $y = \theta(t, x)$, which might not be in $\overline{U}_1$. Finally, the inductive step at the end of the proof is not justified, because the system of ODEs defined by the middle display on page 459 might not be (globally) Lipschitz continuous. Here's how to fix all of these problems:

  (1) Before the statement of Theorem 17.19, insert the following sentence: "If $U \subset \mathbb{R}^n$ is an open subset, a function $F\colon U \to \mathbb{R}^m$ is said to be *locally Lipschitz continuous* if every point $x \in U$ has a neighborhood on which F is Lipschitz continuous. Proposition A.69 shows that every C^1 function is locally Lipschitz continuous." Then insert the following lemma and its proof:

  Lemma. *Suppose $U \subset \mathbb{R}^n$ is an open subset and $F\colon U \to \mathbb{R}^m$ is locally Lipschitz continuous. Then the restriction of F to any compact subset $K \subset U$ is Lipschitz continuous.*

  Proof. For each $x \in K$, there is a positive number $\varepsilon(x)$ such that F is Lipschitz continuous on the ball $B_{2\varepsilon(x)}(x)$, with Lipschitz constant $C(x)$. By compactness, K is covered by finitely many balls $B_{\varepsilon(x_1)}(x_1), \dots, B_{\varepsilon(x_k)}(x_k)$. Let $C = \max\{C(x_1), \dots, C(x_k)\}$, $\delta = \min\{\varepsilon(x_1), \dots, \varepsilon(x_k)\}$, and $M = \sup_K |F|$. For any $x, y \in K$, if $|x - y| < \delta$, then both x and y lie in one of the balls on which F is Lipschitz continuous, so $|F(x) - F(y)| \leq C|x - y|$. On the other hand, if $|x - y| \geq \delta$, then $|F(x) - F(y)| \leq |F(x)| + |F(y)| \leq 2M \leq 2(M/\delta)|x - y|$. □

  (2) Modify the statement of Theorem 17.19 by changing "*Lipschitz continuous*" to "*locally Lipschitz continuous*" in the first sentence.

(3) Replace the first two paragraphs of the proof of Theorem 17.19 by the following three paragraphs:

We will prove the theorem by induction on k. Let $(t_1, x_1) \in J_0 \times U_0$ be arbitrary. It suffices to prove that θ is C^k on some neighborhood of (t_1, x_1). Let J_1 be a bounded open interval containing t_0 and t_1 and such that $\overline{J}_1 \subset J_0$. Because the restriction of θ to $J_0 \times \{x_1\}$ is an integral curve of V, it is continuous and therefore the set $K = \theta(\overline{J}_1 \times \{x_1\})$ is compact. Thus there exists $c > 0$ such that $\overline{B}_{2c}(y) \subset U$ for every $y \in K$. Let $W = \bigcup_{y \in K} B_c(y)$, so that W is a precompact neighborhood of K in U. The restriction of V to $\overline{W}$ is bounded by compactness, and is Lipschitz continuous by the preceding lemma. Let C be a Lipschitz constant for V on $\overline{W}$, and define constants M and T by

$$M = \sup_{\overline{W}} |V|, \qquad T = \sup_{t \in \overline{J}_1} |t - t_0|.$$

For any $x, \widetilde{x} \in W$, both $t \mapsto \theta(t, x)$ and $t \mapsto \theta(t, \widetilde{x})$ are integral curves of V for $t \in J_1$. As long as both curves stay in W, (17.18) implies

$$|\theta(t, x) - \theta(t, \widetilde{x})| \le e^{CT} |\widetilde{x} - x|. \tag{17.19}$$

Choose $r > 0$ such that $2re^{CT} < c$, and let $U_1 = B_r(x_1)$ and $U_2 = B_{2r}(x_1)$. We will prove that θ maps $\overline{J}_1 \times \overline{U}_2$ into W. Assume not, which means there is some $(t_2, x_2) \in \overline{J}_1 \times \overline{U}_2$ such that $\theta(t_2, x_2) \notin W$; for simplicity, assume $t_2 > t_0$. Let τ be the infimum of times $t > t_0$ in J_1 such that $\theta(t, x_2) \notin \overline{W}$. By continuity, this means $\theta(\tau, x_2) \in \partial W$. But because both $\theta(t, x_1)$ and $\theta(t, x_2)$ are in W for $t \in [t_0, \tau]$, (17.19) yields $|\theta(\tau, x_2) - \theta(\tau, x_1)| \le 2re^{CT} < c$, which means that $\theta(\tau, x_2) \in W$, a contradiction. This proves the claim.

For the $k = 0$ step, we will show that θ is continuous on $\overline{J}_1 \times \overline{U}_1$. It follows from (17.19) that it is Lipschitz continuous there as a function of x. We need to show that it is jointly continuous in (t, x).

(4) On the fifth line below equations (17.21), change "nonzero vector $h \in \overline{B}_r(0) \subset \mathbb{R}^n$" to "real number h such that $0 < |h| < r$."

(5) In the last paragraph on page 457, replace the entire third line by "for all $y \in W$ and all $v \in \mathbb{R}^n$ small enough that the line segment from y to $y + v$ is contained in W."

(6) In the second-to-last line of that page, change "$(y, v) \in \overline{U}_1 \times \overline{B}_r(0)$" to just "$(y, v)$."

(7) In the last line of that page, before the sentence beginning with "Because," insert the following sentence: "By compactness, there exists a constant $e > 0$ such that $\overline{B}_e(y) \subset W$ whenever $y \in \theta(\overline{J}_1 \times \overline{U}_1)$."

(8) In the second line of page 458, replace "such that ..." by "such that $\delta < e$ and ...."

(9) In inequality (17.24), replace $\overline{U}_1$ by $\overline{W}$.

(10) In the text line between the two multi-line displays on page 458, change "nonzero $h, \widetilde{h} \in \overline{B}_r(0)$" to "sufficiently small nonzero real numbers h and $\widetilde{h}$."

(11) In the first line of the last paragraph on page 458, change $\delta \le r$ to $\delta < e$.

(12) In the second line of that paragraph, change $\overline{U}_1$ to $\overline{W}$.

- **Page 457, second line of the last paragraph:** Change "give" to "gives."
- **Page 458, second multi-line equation:** In the middle line of the equation, change $(\Delta_h)^k_j(x)$ to $(\Delta_h)^k_j(t, x)$.
- **Page 461, line 10:** Delete repeated "the."
- **Page 466, second displayed equation:** Delete the superfluous right parenthesis in the numerator of the fraction on the right-hand side.

- **Page 468, statement of Lemma 18.4:** Replace the last sentence and the accompanying diagram by the following: "*If X and Y are F-related, then for each $t \in \mathbb{R}$, $F(M_t) \subset N_t$ and $\eta_t \circ F = F \circ \theta_t$ on M_t:*

$$\begin{tikzcd} M_t \arrow[r, "F"] \arrow[d, "\theta_t"'] & N_t \arrow[d, "\eta_t"] \\ M \arrow[r, "F"'] & N. \end{tikzcd}$$

Conversely, if for each $p \in M$ there is an $\varepsilon > 0$ such that $\eta_t \circ F(p) = F \circ \theta_t(p)$ for all $|t| < \varepsilon$, then X and Y are F-related.

- **Page 469, statement of Proposition 18.5(f):** Replace the statement by the following: "*For each $p \in M$, if either $\theta_t \circ \psi_s(p)$ or $\psi_s \circ \theta_t(p)$ is defined for all $(s,t) \in J \times K$, where J and K are open intervals containing 0, then both are defined and they are equal.*"

- **Page 469, last line:** Change "(e) implies (b)" to "(e) implies (c)."

- **Page 470, three lines below (18.5):** Change "(b) implies (e)" to "(c) implies (e)."

- **Page 470, last paragraph:** Replace the two sentences beginning with "Thus Lemma 18.4 applied with ..." by the following: "Thus Lemma 18.4 applied with $F = \psi_s \colon M_s \longrightarrow M_{-s}$, $X = V|_{M_s}$, and $Y = V|_{M_{-s}}$ implies that $\theta_t \circ \psi_s(p) = \psi_s \circ \theta_t(p)$ provided that $\theta_\tau(p) \in M_s$ for all τ in some interval containing 0 and t. (This requirement ensures that p is in the time-t flow of $V|_{M_s}$, which might not be the same as the intersection of M_s with the domain of θ_t.) Since (e) implies (d), the same argument with V and W reversed shows that this also holds when $\psi_\sigma(p)$ is in the domain of θ_t for σ in an interval containing 0 and s."

- **Page 471, proof of Theorem 18.6, third paragraph, last two sentences:** Replace these parenthesized sentences by "(Just choose $\varepsilon_k > 0$ and $U_k \subset U$ such that θ_k maps $(-\varepsilon_k, \varepsilon_k) \times U_k$ into U, and then inductively choose ε_i and U_i such that θ_i maps $(-\varepsilon_i, \varepsilon_i) \times U_i$ into U_{i+1}. Taking $\varepsilon = \min\{\varepsilon_i\}$ and $W = U_1$ does the trick.)"

- **Page 472, second line:** Change $u_0 \in W$ to $u_0 \in (-\varepsilon, \varepsilon)^k \times S$.

- **Page 472, second displayed equation:** Replace V_p by $V_i|_p$, so the equation becomes

$$\psi_* \left.\frac{\partial}{\partial u^i}\right|_0 = V_i|_p.$$

- **Page 472, line 5 from the bottom:** Change "coordinate submanifold" to "coordinate submanifold S."

- **Page 473, Example 18.7:** Change the last two displayed equations in the example to

$$\psi(u,v) = \eta_v \circ \theta_u(1,0) = (e^v \cos u, e^v \sin u)$$

and

$$(u,v) = \left(\tan^{-1}(y/x), \log\sqrt{x^2+y^2}\right)$$

(so that the coordinate vector fields $\partial/\partial u$, $\partial/\partial v$ will correspond to V and W, respectively, instead of the other way around).

- **Page 481, second paragraph after the statement of Theorem 18.19:** Replace the second sentence of this paragraph by the following: "The proof we will give was discovered in 1971 by Alan Weinstein [Wei71], based on a technique due to Jürgen Moser [Mos65]."

- **Page 482, lines 3 and 7 from the bottom:** Change "Lemma" to "Proposition."
- **Page 484, last three lines of the proof of the Darboux theorem:** Replace the last two sentences of the proof with "It follows from Problem 17-15(b) that $\theta_{1,0}$ is a diffeomorphism onto its image, so it is the desired coordinate map." (The argument given in the text is not incorrect, but this one is simpler.)
- **Page 487, line 18:** Change "electromagnetic" to "electrostatic."
- **Page 488, first displayed equation:** Change v_i to v^i.
- **Page 488, two lines below equation (18.22):** Change T^*M to T^*Q.
- **Page 488, three lines below equation (18.22):** Change $E(p,q)$ to $E(q,p)$.
- **Page 490, proof of Theorem 18.27, second paragraph:** In the second line of the paragraph, change "by Exercise 18.6" to "by definition."
- **Page 492, Problem 18-6(a):** Change $X \in \mathcal{T}^0(M)$ to $f \in \mathcal{T}^0(M)$.
- **Page 492, Problem 18-9:** Replace the first two sentences in the problem statement by the following: "Prove the following global version of the Darboux theorem, due to Moser [Mos65]: Let M be a compact manifold, and let $\{\omega_t : t \in [0,1]\}$ be a family of symplectic forms on M. Suppose there exists a smooth function $f\colon [0,1] \times M \longrightarrow \mathbb{R}$ such that $\omega_t = \omega_0 + df_t$ for each $t \in [0,1]$, where $f_t(x) = f(t,x)$."
- **Page 495, line 14 from the bottom:** Change "An immersed submanifold" to "A nonempty immersed submanifold."
- **Page 497, last line:** After the period, insert the following sentence: "The assumption (19.2) together with the fact that D is k-dimensional implies that the forms $\omega^1, \dots, \omega^{n-k}$ are independent on U for dimensional reasons."
- **Page 500, proof of Proposition 19.9, second line:** Change "defined on an open set $U \subset M$" to "that annihilates D on an open set $U \subset M$."
- **Page 501, proof of Theorem 19.10, first paragraph:** Insert "independent" before the word "smooth" (twice). [Remark: This change is not logically necessary, but it clarifies why Theorem 18.6 applies in this case.]
- **Page 504, line 2:** In this displayed equation, change z to w twice:

$$\psi(u,v,w) = \alpha_u \circ \beta_v(0,0,w) = \alpha_u(0,v,w) = (u,v,w+uv).$$

- **Page 504, line 4:** Change z to w, so it reads $\psi(u,v,w) = (u,v,w+uv)$.
- **Page 505, proof of Proposition 19.13, fourth line from end:** Replace $F\colon N \longrightarrow H$ by $F\colon M \longrightarrow H$.
- **Page 507, line 11 from the bottom:** Change $\mathbb{R}^n$ to $\mathbb{R}^3$.
- **Page 512, line 8 from the bottom:** Change "Lemma 19.12" to "Proposition 19.12."
- **Page 513, first full paragraph, last line:** Change "containing q" to "containing q and q'."
- **Page 514, first full paragraph, next-to-last line:** Change $\psi' \circ \psi$ to $\psi' \circ \psi^{-1}$.
- **Page 515, Problem 19-1:** An extra compatibility condition has to be added to the definition of a Pfaffian system for the result to be true: For each $\alpha, \beta \in A$, the restrictions to $U_\alpha \cap U_\beta$ of $\left(\omega_\alpha^1, \dots, \omega_\alpha^{n-k}\right)$ and $\left(\omega_\beta^1, \dots, \omega_\beta^{n-k}\right)$ generate the same ideal in $\mathcal{A}^*(U_\alpha \cap U_\beta)$.

- **Page 517, Problem 19-11:** Change $x > 0$ to $y > 0$ in the definition of U.
- **Page 517, Problem 19-15, third line:** Change $p \in S$ to $p \in N$.
- **Page 526, statement of Proposition 20.9:** The range of the function Z should be $\mathfrak{g}$, not $\mathbb{R}$.
- **Page 526, proof of Proposition 20.9:** In the displayed equation on line 8 of the proof, the last G should be $\mathfrak{g}$.
- **Page 534, lines 1 and 2:** Change π_1 to π_2 (twice).
- **Page 535, proof of Lemma 20.18:** The last two sentences in this proof should be in reverse order, since the induction mentioned in the next-to-last sentence doesn't work unless you know $ghg^{-1} \in H$ for all $g \in V$ and all $h \in H$. Leave the words "Since" and "Similarly" where they are, but otherwise interchange the two last sentences of the proof (with appropriate minor grammatical and punctuation changes).
- **Page 537, proof of Theorem 20.21:** In the next-to-last line, change "Proposition 9.26" to "Proposition 9.29(b)." Also, Problem 9-27 is not sufficient to prove the the centrality of $\operatorname{Ker}\Phi$; you will need to prove the following fact: *If a connected topological group acts continuously on a discrete space, then the action is trivial.*
- **Page 544, last line:** Insert "in" before "the following lemma."
- **Page 552, statement of Lemma A.17:** In part (e), replace "*compact subsets*" by "*disjoint compact subsets.*"
- **Page 556, second line in the "Covering Maps" section:** Change "path connected and locally path connected" to "connected and locally path connected." After that sentence, insert the following: "It follows from Lemma A.16(c) that $\widetilde{X}$ is path connected."
- **Page 568, first full paragraph:** Change the second sentence to "By choosing integers $1 \le i_1 < \cdots < i_k \le m$ and $1 \le j_1 < \cdots < j_l \le n$, we obtain . . . ."
- **Page 572, Exercise A.44:** Change "by a matrix" to "by a certain matrix" (twice).
- **Page 573, line below equation (A.7):** Change $1 \times n$ to $1 \times (n-1)$.
- **Page 577, Exercise A.50:** Change "basis map" to "basis isomorphism."
- **Page 585, statement of Corollary A.53, second line:** Change f to F.
- **Page 592, Proposition A.63(b), third line:** Change "for each i, j" to "for each $i \ne j$."
- **Page 599, just after reference [War83]:** Insert the following reference:
  [Wei71] Alan Weinstein. Symplectic manifolds and their Lagrangian submanifolds. *Adv. Math.*, 6:329–346, 1971.
- **Page 602, left column:** Insert the following index entries after $\mathcal{B}_p(M)$:

  $B_r(x)$ (open ball in a metric space), 542
  $\overline{B}_r(x)$ (closed ball in a metric space), 542
- **Page 623, right column, line 3 from the bottom:** The last index entry for "smooth/structure/ uniqueness of" should refer to page 461, not 460.
- **Page 627, right column, after line 1:** Insert the entry "universal covering manifold, 41."