Math 300 **Introduction to Mathematical Reasoning** **Autumn 2018**

Handout 10: W-Credit Assignment

This is an optional assignment for those who would like to try for W credit. The first draft is due in class on Friday, November 30, and the final draft is due at the final exam, 8:30AM on Thursday, December 13.

The assignment will be graded on a 100 point scale, according to the following criteria:

- Turning in the first draft with all instructions followed correctly: 50 points
- Final draft, mathematical correctness: 25 points
- Final draft, expository effectiveness: 25 points

In order to get W credit for this course, you need to earn at least 75 points on this assignment.

Here's the assignment:

Imagine that our textbook author has decided that the book should include more discussion about the fact that injectivity and surjectivity of a function are highly sensitive to its domain and codomain. You have been assigned to write a new section for the book, to come immediately after Section 6.3. Your discussion should be written textbook-style, with complete sentences organized in paragraphs, and it should contain an introduction, motivations, definitions, intuitive explanations, theorems, proofs, and a conclusion. Your writing should follow all the guidelines of Handout 6, and every mathematical claim you make should be stated as a theorem and carefully proved. You may use and refer to any theorem in the book prior to Section 6.4, and any theorem on course handouts 1–9.

You should begin by reviewing the relevant definitions, and reminding the reader of their intuitive meanings. Then you're going to focus on two particular formulas for defining functions between subsets of the real line.

The first one is a family of functions f_1, f_2, f_3, all of which are given by the formula

$$f_i(x) = \frac{1+x}{1-x}.$$

Begin by defining $f_1 \colon \mathbb{R} - \{1\} \to \mathbb{R}$ by this formula, and determine whether this function is injective, surjective, bijective, or none of the above; and if it is not surjective, determine its range. (And, of course, prove your answers correct.) Next, define another function $f_2 \colon (-1,1) \to (0,\infty)$ by the same formula, and answer the same questions. Finally, determine two other intervals $I_1, I_2 \subseteq \mathbb{R}$ such that the function $f_3 \colon I_1 \to I_2$ is bijective.

Then you should focus on functions g_1, g_2, g_3, all given by the formula

$$g_i(x) = x^2 - 2x.$$

Answer the same questions for the functions given by that formula, with the following domains and codomains:

$$g_1 \colon \mathbb{R} \to \mathbb{R},$$
$$g_2 \colon (0,\infty) \to [-1,\infty),$$
$$g_3 \colon I_1 \to I_2,$$

where you choose intervals I_1 and I_2 such that g_3 is bijective.