Math 441 **Topology** **Fall 2012**

Assignment #4 Supplement

Problem S2: Let X be a set, and let $\mathscr{T}_1$ and $\mathscr{T}_2$ be topologies on X. Let X_1 and X_2 denote the topological spaces $(X, \mathscr{T}_1)$ and $(X, \mathscr{T}_2)$, respectively, and let $i\colon X_1 \to X_2$ denote the identity map of X.

(a) Show that i is continuous from X_1 to X_2 if and only if $\mathscr{T}_1$ is finer than $\mathscr{T}_2$.
(b) Show that $i\colon X_1 \to X_2$ is a homeomorphism if and only if $\mathscr{T}_1 = \mathscr{T}_2$.

Problem S3: Suppose X and Y are topological spaces and $f\colon X \to Y$ is a continuous map.

(a) If f has a continuous left inverse $g\colon Y \to X$, prove that f is a topological embedding.
(b) Let $f\colon \mathbb{R} \smallsetminus \{0\} \to \mathbb{R}$ be the function $f(x) = 1/x$. Prove that f is a topological embedding that does not have a continuous left inverse.