Math 442 **Differential Geometry** **Winter 2011**

Assignment #1: Due 1/14/11

Reading:

- Skim all of Chapter 1.
- Read carefully Section 2.2.
- Look quickly through Section 2.3.

Written Assignment:

A. Show that a composition of two Euclidean motions is a Euclidean motion, and the inverse of a Euclidean motion is a Euclidean motion, by proving that

$$F_{A,b} \circ F_{B,c} = F_{AB,Ac+b} \qquad \text{and} \qquad (F_{A,b})^{-1} = F_{A^{-1},-A^{-1}b}.$$

(See pp. 14–15 for the definition of a Euclidean motion. If $F\colon \mathbb{R}^n \to \mathbb{R}^n$ is any bijective map, its ***inverse*** is the unique map $F^{-1}\colon \mathbb{R}^n \to \mathbb{R}^n$ that satisfies $F \circ F^{-1}(x) = x$ and $F^{-1} \circ F(x) = x$ for all $x \in \mathbb{R}^n$.)

B. Suppose $c\colon [a,b] \to \mathbb{R}^n$ is a parametrized curve segment. If $F_{A,v}\colon \mathbb{R}^n \to \mathbb{R}^n$ is any Euclidean motion, show that $I[F_{A,v} \circ c] = I[c]$, where I denotes the integral length of a curve:

$$I[c] = \int_a^b \|\dot{c}(t)\| dt.$$

C. Bär, Exercise 2.2 (page 28).

D. For each of the following parametrized curves, write down an explicit orientation-preserving reparametrization by arc-length. Be sure to specify the domain of the new parametrization.

(a) $c_1\colon [0,2\pi] \to \mathbb{R}^2$ is given by $c_1(t) = (3\cos t, 3\sin t)$.

(b) $c_2\colon (0,\infty) \to \mathbb{R}^2$ is given by $c_2(t) = (t^2, t^3)$.

E. Bär, Exercise 2.5 (page 33).

F. Bär, Exercise 2.7 (page 34).