Math 442 **Differential Geometry** **Winter 2011**

Assignment #5: Due 2/7/11

Reading:

- Section 3.2 (up through page 95 only).

Written Assignment:

These problems will not be collected for grading. Be prepared to discuss as many of these as possible in class on Monday, Feb. 7.

A. Suppose $C \subset \mathbb{R}^2$ is a regular 1-manifold. The ***generalized cylinder*** determined by C is the set $S = C \times \mathbb{R} \subset \mathbb{R}^3$, that is,
$$S = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in C\}.$$
Show that every generalized cylinder is a regular surface.

B. When C is the unit circle in $\mathbb{R}^2$ centered at the origin, the generalized cylinder S determined by C is usually just called ***the cylinder***:
$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}.$$
Determine a basis for the tangent plane T_pS at an arbitrary point $p = (x_0, y_0, z_0) \in S$.

C. Let $S_2 \subset \mathbb{R}^3$ be the following cone:
$$S_2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2,\ z > 0\}.$$
Show that S_2 is a regular surface, and find two local parametrizations that cover the whole surface. Determine a basis for the tangent plane T_pS_2 at an arbitrary point $p = (x_0, y_0, z_0) \in S_2$.

D. Define $f\colon \mathbb{R}^3 \to \mathbb{R}$ by $f(x, y, z) = (x - y)^2$, and let $S_3 = f^{-1}(0)$. Prove that S_3 is not a regular level set of f, and yet it is a regular surface. Determine a basis for the tangent plane T_pS_3 at an arbitrary point $p = (x_0, y_0, z_0) \in S_3$.

E. Let $U \subset \mathbb{R}^2$ be an open set, and let $S_4 = U \times \{0\} = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in U\}$. Let $f\colon U \to \mathbb{R}$ be a smooth function, and let S_5 be the graph of f:
$$S_5 = \Gamma(f) = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in U,\ z = f(x, y)\}.$$
Prove that S_4 and S_5 are diffeomorphic to each other. Determine a basis for the tangent plane T_pS_5 at an arbitrary point $p = (x_0, y_0, z_0) \in S_5$.