Math 442 **Differential Geometry** **Winter 2011**

Assignment #6: Due 2/18/11
(CORRECTED VERSION)

Reading:

- Bär, Sections 3.2 (pp. 96–98), 3.3, 3.4.

Written Assignment:

A. Bär, exercise 3.5 (page 102).

B. Bär, exercise 3.6 (page 102).

C. Bär, exercise 3.7 (page 102).

D. Bär, exercise 3.8 (page 105).

E. Compute the matrix of the first fundamental form of the unit sphere with respect to the stereographic projection parametrization F_1 described in the hint to problem 3.1.

F. For the surface parametrizations of exercises 3.6 and 3.7 above (the graph of a function and a cone), compute a continuous unit normal field N as a function of the parameters (x, y).

G. The Möbius strip is described informally in Example 3.4.5. Formally, it can be defined as the surface $S \subset \mathbb{R}^3$ that is the image of the map $F\colon \mathbb{R} \times (-1, 1) \to \mathbb{R}^3$ defined by

$$F(t,s) = \Big(\cos t + s \cos t \cos \frac{t}{2},\ \sin t + s \sin t \cos \frac{t}{2},\ s \sin \frac{t}{2} \Big).$$

Let $U_1, U_2 \subset \mathbb{R}^2$ be the sets $U_1 = (0, 2\pi) \times (-1, 1)$ and $U_2 = (-\pi, \pi) \times (-1, 1)$, and let $F_1 = F|_{U_1}$, $F_2 = F|_{U_2}$. Then F_1 and F_2 are regular surface parametrizations whose images cover all of S. (You don't need to prove this.)

(a) For each $i = 1, 2$, let N_i denote the unit normal field determined by the parametrization F_i as explained on page 105 of Bär. Prove that if there exists a continuous unit normal field N on all of S, then on the image of F_1, then either N agrees everywhere with N_1 or N agrees everywhere with $-N_1$. Similarly, show that on the image of F_2, N agrees everywhere with N_2 or with $-N_2$. [Hint: You don't need to compute the general formula for N_1 and N_2. For each $i = 1, 2$, consider the function $f_i(t, s) = \big\langle N(F(t, s)), N_i(F(t, s))\big\rangle$.]

(b) Compute both N_1 and N_2 at the points $(0, 1, 0)$ and $(0, -1, 0)$.

(c) Prove that there does not exist a continuous unit normal field on S, and therefore S is not orientable.