Math 442 **Differential Geometry** **Winter 2011**

Assignment #8: Due 3/4/11 (CORRECTED VERSION 3)

Reading:

- Bär, Sections 3.7 and 3.8.

Written Assignment:

A. Bär, Exercise 3.12 (p. 126).

B. Let $k > 0$, and let $S \subset \mathbb{R}^3$ be the graph of the function $g(x, y) = kxy$ (called a ***hyperbolic paraboloid***). Compute the Gauss and mean curvatures of S at the origin.

C. Let $H \subset \mathbb{R}^2$ be the right half-plane, and let $c\colon (-\pi/2, \pi/2) \to H$ be the parametrized curve given by $c(t) = (r(t), s(t))$, where

$$r(t) = \tfrac{1}{2}\cos t,$$
$$s(t) = \int_0^t \sqrt{1 - \tfrac{1}{4}\sin^2 u}\, du.$$

Let C be the image of c, and let S_C be the surface of revolution with generating curve C. You may use without proof the fact that the image of C is a regular 1-manifold and thus S_C is a regular surface. Show that S_C has Gauss curvature identically equal to 1, but that its principal curvatures are not constant.

D. Suppose $S \subset \mathbb{R}^3$ is a regular surface with positive Gauss curvature everywhere. Suppose $c\colon I \to \mathbb{R}^3$ is a unit-speed curve whose image lies in S. Recall that the ***curvature of c*** is defined as $\kappa(t) = \|\ddot{c}(t)\|$ (Def. 2.3.2).

(a) Show that the curvature of c satisfies $\kappa(t) \geq \min(|\kappa_1|, |\kappa_2|)$, where κ_1 and κ_2 are the principal curvatures of S at $c(t)$.

(b) Still assuming that S has positive Gauss curvature everywhere, is it necessarily true that $\kappa(t) \leq \max(|\kappa_1|, |\kappa_2|)$? Either prove it or give a counterexample.

(c) If S does not have positive Gauss curvature everywhere, is it necessarily true that $\kappa(t) \geq \min(|\kappa_1|, |\kappa_2|)$? Either prove it or give a counterexample.