Math 442 **Differential Geometry** **Winter 2013**

Assignment #4

Supplementary Exercises (CORRECTED)

S8. Let $I \subseteq \mathbb{R}$ be an open interval, and let $\sigma\colon I \to \mathbb{R}^3$ be a smooth regular curve. Suppose $t_0 \in I$ is a point where $\|\sigma(t)\|$ attains its maximum value. Prove that $\kappa(t_0) \geq 1/\|\sigma(t_0)\|$.

S9. Let a and b be real numbers such that $0 < a < b$, and let $\sigma\colon \mathbb{R} \to \mathbb{R}^2$ be the plane curve $\sigma(t) = (a\cos t, b\sin t)$. (Its trace is an ellipse with major axis $2b$ and minor axis $2a$. Note that this parametrization is not necessarily unit-speed.) Find the maximum and minimum values of the oriented curvature of σ in terms of a and b.

S10. Suppose $I \subseteq \mathbb{R}$ is an interval, $s_0 \in I$, and $f\colon I \to \mathbb{R}$ is a smooth function. Define $\theta\colon I \to \mathbb{R}$ and $\sigma\colon I \to \mathbb{R}^2$ by

$$\theta(s) = \int_{s_0}^{s} f(u)\,du,$$

$$\sigma(s) = \left(\int_{s_0}^{s} \cos\theta(u)\,du,\ \int_{s_0}^{s} \sin\theta(u)\,du \right).$$

Prove that σ is a unit-speed curve whose oriented curvature is $\tilde{\kappa}(s) = f(s)$.