Math 442 **Differential Geometry** **Winter 2013**

Assignment #5

Supplementary Exercises (CORRECTED)

S11. Suppose $\varphi\colon [a,b] \to S^1$ is a continuous closed curve in the circle, and $\tilde\varphi\colon [c,d] \to S^1$ is a forward reparametrization of φ. Show that $\deg\varphi = \deg\tilde\varphi$.

S12. Suppose $\sigma\colon [a,b] \to \mathbb{R}^2$ is a continuous closed plane curve and p is a point in $\mathbb{R}^2$ that is not in the trace of σ.

(a) If $\tilde\sigma\colon [c,d] \to \mathbb{R}^2$ is a forward reparametrization of σ, show that $\iota_p(\sigma) = \iota_p(\tilde\sigma)$.

(b) If $\rho\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a rigid motion, show that $\iota_{\rho(p)}(\rho\circ\sigma) = \iota_p(\sigma)$.

S13. Suppose $\sigma\colon [0,2\pi] \to \mathbb{R}^2$ is a smooth simple closed plane curve. We don't know the formula for σ, but we're given that

$$\sigma'(s) = \big(\sin(s+\cos 2s),\ \cos(s+\cos 2s)\big) \qquad \text{for all } s \in [0,2\pi].$$

Compute the following:

(a) The length of σ.

(b) The oriented curvature of σ.

S14. Suppose $\sigma\colon I \to \mathbb{R}^2$ is a smooth unit-speed plane curve, and $\theta\colon I \to \mathbb{R}$ is a smooth function such that $\sigma'(s) = (\cos\theta(s), \sin\theta(s))$ for all $s \in I$. Show that the oriented curvature of σ is $\tilde\kappa(s) = \theta'(s)$.