Math 443 **Topics in Topology and Geometry** **Spring 2003**

Assignment #6 (CORRECTED): Due Wednesday, 5/21/03

I. Reading:

- Reread do Carmo, Section 4-5.

II. Required problems:

1. do Carmo, Section 4-5 (pp. 282–283), #5. *(The problem should be rephrased as follows.)*
   Let C be a parallel of colatitude φ on the unit sphere S^2, oriented by the outward normal, and let w_0 be a unit vector tangent to C at a point $p \in C$ (cf. Example 1, Sec. 4-4). Take the parallel transport of w_0 along C, and let $\Delta\theta$ be the angle between w_0 and the value of its parallel transport after a complete turn. Show directly (not by invoking the formula on p. 271) that $\Delta\theta = 2\pi(1 - \cos\varphi)$, and that

   $$\lim_{\varphi \to 0} \frac{\Delta\theta}{A} = 1 = \text{curvature of } S^2,$$

   where A is the area of the region R of S^2 bounded by C.

2. If $S \subset \mathbb{R}^3$ is a regular surface, a *geodesic polygon* in S is a simple region $R \subset S$ whose regular arcs are all segments of geodesics.
   (a) Give an example of a geodesic polygon with no vertices.
   (b) Give an example of a geodesic polygon with exactly two vertices.
   (c) If S has everywhere nonpositive Gaussian curvature, show that every geodesic polygon in S has at least three vertices.