Math 443 **Differential Geometry** **Spring 2012**

Assignment #2: Due Friday, 4/13/12 (UPDATED)

Reading:

- Page 116 (just read and understand the *statement* of Theorem 4.16).
- Pages 118–120 (Remark 4.20 through Remark 4.22).
- Section 7.1.

Written Assignment:

A. Suppose S is a connected surface. Prove that if either of the following is true, then S is orientable:

(a) The mean curvature of S is nowhere zero.

(b) The Gauss curvature of S is everywhere positive.

B. Prove the following corollary to Theorem 3.47: *Suppose S is an oriented surface with positive Gauss curvature. Let the principal curvatures of S be denoted by k_1, k_2 with $k_1 \leq k_2$. If there is a point $p_0 \in S$ where k_1 achieves its minimum and k_2 achieves its maximum, then S is totally umbilical.*

C. Exercise 3.50 (page 94).

D. Exercise 3.51 (page 94).

E. Exercise (7) (page 95).

F. Suppose $H = \{(u, v) : u > 0\} \subset \mathbb{R}^2$ is the right half-plane, and $C \subset H$ is a simple curve, and $\alpha\colon I \to H$ is a local parametrization of C (not necessarily unit speed), written in the form $\alpha(t) = (a(t), b(t))$. Let $S_C \subset \mathbb{R}^3$ be the surface of revolution generated by C, and let $X\colon I \times \mathbb{R} \to \mathbb{R}^3$ be the map

$$X(t, \theta) = (a(t)\cos\theta,\ a(t)\sin\theta,\ b(t)).$$

Then X, restricted to any rectangle of the form $I \times (k, k + 2\pi)$, is a local parametrization of S_C (you don't have to prove this either). Compute the principal curvatures, Gauss curvature, and mean curvature of S_C in terms of this parametrization. (In other words, compute $k_i \circ X$, $K \circ X$, and $H \circ X$ as functions of (t, θ).)

G. Let H be the right half-plane. For each of the following maps $\alpha\colon \mathbb{R} \to H$, the image $\alpha(\mathbb{R})$ is a simple curve in H (you don't have to prove this). Compute the Gauss and mean curvatures of the corresponding surfaces of revolution.

(a) $I = \mathbb{R}$, $\alpha(t) = (2 + \cos t, \sin t)$ (this generates a torus of revolution).

(b) $I = \mathbb{R}$, $\alpha(t) = (\cosh t, t)$ (this generates a surface called a *catenoid*).

H. ~~Let H be the right half-plane, and let $\alpha\colon (-\pi/2, \pi/2) \to H$ be the parametrized curve given by $\alpha(t) = (a(t), b(t))$, where~~

$$a(t) = \tfrac{1}{2}\cos t,$$
$$b(t) = \int_0^t \sqrt{1 - \tfrac{1}{4}\sin^2 u}\, du.$$

~~If we let C be the image of α, then C is a simple curve (you don't have to prove this). Show that the surface of revolution S_C has Gauss curvature identically equal to 1, but that its principal curvatures are not constant.~~

I. ~~Let C be a simple curve in the right half-plane that admits a global parametrization, and let S_C be the surface of revolution it generates. Prove that S_C has identically zero Gauss curvature if and only if C is contained in a straight line.~~