**Math 443** **Differential Geometry** **Spring 2012**

**Assignment #3: Due Friday, 4/20/12**

**Reading:**

- Sections 7.2, 7.3. (Skip 7.4.)
- Skim Section 7.5.

**Written Assignment:**

A. Let $H$ be the right half-plane, and let $\alpha\colon (-\pi/2, \pi/2) \to H$ be the parametrized curve given by $\alpha(t) = (a(t), b(t))$, where

$$a(t) = \tfrac{1}{2}\cos t,$$
$$b(t) = \int_0^t \sqrt{1 - \tfrac{1}{4}\sin^2 u}\, du.$$

If we let $C$ be the image of $\alpha$, then $C$ is a simple curve (you don't have to prove this). Show that the surface of revolution $S_C$ has Gauss curvature identically equal to 1, but that its principal curvatures are not constant.

B. Let $C$ be a simple curve in the right half-plane that admits a global parametrization, and let $S_C$ be the surface of revolution it generates. Prove that $S_C$ has identically zero Gauss curvature if and only if $C$ is contained in a straight line.

C. Exercise 7.4 (page 208).

D. Exercise (2) (page 239).

E. Exercise (4) (page 239).

F. Let $S, S' \subset \mathbb{R}^3$ be the following surfaces:

$$S = \big\{(x,y,z) : x^2 + y^2 > 0,\ z = 0\big\} \qquad \text{(the } xy\text{-plane with the origin removed),}$$
$$S' = \big\{(x,y,z) : x^2 + y^2 = \tfrac{1}{3}z^2,\ z > 0\big\} \qquad \text{(the upper half of a cone).}$$

Define a map $f\colon S \to \mathbb{R}^3$ by

$$f(x,y,0) = \frac{1}{2\sqrt{x^2+y^2}}\Big(x^2 - y^2,\ 2xy,\ \sqrt{3}(x^2+y^2)\Big).$$

Show that $f$ is a local isometry from $S$ to $S'$. Is it a global isometry?