**Math 443** **Differential Geometry** **Spring 2012**

**Assignment #8: Due Friday, 6/1/12**

**Written Assignment:**

A. Shifrin, Exercise 3.1 #3, page 89.

B. Suppose $S \subset \mathbb{R}^3$ is a surface with boundary such that $S$ is diffeomorphic to a closed disk. Suppose further that its boundary curve is a closed geodesic. Prove that the Gauss curvature of $S$ must be positive somewhere.

C. Suppose $S \subset \mathbb{R}^3$ is a surface with boundary such that $S$ is diffeomorphic to the cylinder $S^1 \times [0, 1]$. Suppose further that both boundary curves are closed geodesics. If the Gauss curvature of $S$ is not identically zero, prove that it attains both positive and negative values.

D. Let $S$ be the paraboloid defined by $z = x^2 + y^2$, and for each $r > 0$, let $S_r$ be the portion of $S$ where $z \leq r$. Verify the Gauss–Bonnet formula for $S_r$ by computing each term separately.