Axioms of Incidence Geometry

Incidence Axiom 1. *For every pair of distinct points P and Q there is exactly one line ℓ such that P and Q lie on ℓ.*

Incidence Axiom 2. *For every line ℓ there exist at least two distinct points P and Q such that both P and Q lie on ℓ.*

Incidence Axiom 3. *There exist three points that do not all lie on any one line.*

Theorems of Incidence Geometry

Theorem 3.6.1. *If ℓ and m are distinct, nonparallel lines, then there exists a unique point P such that P lies on both ℓ and m.*

Theorem 3.6.2. *If ℓ is any line, then there exists at least one point P such that P does not lie on ℓ.*

Theorem 3.6.3. *If P is any point, then there are at least two distinct lines ℓ and m such that P lies on both ℓ and m.*

Theorem 3.6.4. *If ℓ is any line, then there exist lines m and n such that ℓ, m, and n are distinct and both m and n intersect ℓ.*

Theorem 3.6.5. *If P is any point, then there exists at least one line ℓ such that P does not lie on ℓ.*

Theorem 3.6.6. *There exist three distinct lines such that no point lies on all three of the lines.*

Theorem 3.6.7. *If P is any point, then there exist points Q and R such that P, Q, and R are noncollinear.*

Theorem 3.6.8. *If P and Q are two points such that $P \neq Q$, then there exists a point R such that P, Q, and R are noncollinear.*