CORRECTIONS TO
Axiomatic Geometry
BY JOHN M. LEE
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(7/17/09) **Page 9, four lines above the section heading:** Change "Proposition I.10" to "Proposition I.16."

(7/27/09) **Page 9, last line:** Change "comprise" to "constitute."

(8/14/09) **Page 11, first paragraph, last line:** "because" is misspelled.

(7/27/09) **Page 13, next to last line:** Change "*exterior angle theorem*" to "*exterior angle inequality.*"

(7/16/09) **Page between Chapters 1 and 2:** This page should be numbered 16, not 18.

(7/6/09) **Page 21, last line:** Change $\{5, 6, 7\}$ to $\{1, 5, 6\}$. [Alexei]

(7/10/09) **Page 26, displayed equations just below (2.1):** This line should be numbered (2.2). Also, the numerator of the first equation should be $y_2 - y_1$, not $y_2 - y_2$. Thus the equation should read

$$A = \frac{y_2 - y_1}{x_2 - x_1}, \qquad B = -1, \qquad C = y_1 - \left(\frac{y_2 - y_1}{x_2 - x_1}\right) x_1. \qquad (2.2)$$

(7/10/09) **Page 26, middle of the page:** Replace the paragraph starting "To prove uniqueness" with the following:

To prove uniqueness, suppose there is another equation $A'x + B'y + C' = 0$ satisfied by both (x_1, y_1) and (x_2, y_2), with A' and B' not both zero. The basic idea for proving uniqueness is that both lines have to have the same slope, namely $(y_2 - y_1)/(x_2 - x_1)$; we will use this to show that one equation is a nonzero multiple of the other, i.e., that there is a nonzero constant k such that $A' = kA$, $B' = kB$, and $C' = kC$.

Our assumption means that the following two equations are satisfied:

$$A'x_1 + B'y_1 + C' = 0,$$
$$A'x_2 + B'y_2 + C' = 0.$$

Subtracting the first equation from the second, we obtain $A'(x_2 - x_1) + B'(y_2 - y_1) = 0$, which implies

$$A' = -B'\frac{y_2 - y_1}{x_2 - x_1}. \qquad (2.3)$$

If B' were zero, this would imply $A' = 0$, which we are assuming is not the case; thus $B' \neq 0$.

We can also do exactly the same computation with the equation $Ax + By + C = 0$ that we constructed earlier, where A, B, C are defined by (2.2). This leads to

$$A = -B\frac{y_2 - y_1}{x_2 - x_1}, \qquad (2.4)$$

from which we can also conclude that $B \neq 0$.

Now let $k = B'/B$. Then obviously $B' = kB$, and equations (2.3) and (2.4) together yield $A' = kA$. Since (x_1, y_1) satisfies both equations, we have

$$Ax_1 + By_1 + C = 0,$$
$$kAx_1 + kBy_1 + C' = 0.$$

Multiplying the first equation by k and subtracting the second, we obtain $kC - C' = 0$, or $C' = kC$. This completes the proof that our new equation is a nonzero constant multiple of the original one, so the two equations have the same solution set and therefore represent the same line.

(7/20/09) **Page 33, next to last line:** Change "the point" to "that point." [Amy]

(7/19/09) **Page 37, fourth paragraph:** Delete redundant "subject." [Neesha]

(7/16/09) **Page 41, two lines above Postulate 5:** Change "it said" to "it is said."

(7/20/09) **Page 41, last paragraph, first line:** Change "any function" to "any bijective function." [Lindsay]

(7/22/09) **page 43, proof of Theorem 4.8, last sentence:** Change that sentence to "Because f is a function, $P = Q$ implies $f(P) = f(Q)$; and because f is injective, the converse holds as well. This proves (b) and (c)."

(7/17/09) **Page 45, statement of Corollary 4.15:** Change $f(A) * f(B) * f(B)$ to $f(A) * f(B) * f(C)$. [Cindy]

(7/17/09) **Page 50, displayed equation (5.2):** Change $f(A) < f(B)$ to $f(A) > f(B)$. [Cindy]

(7/17/09) **Page 50, proof of Lemma 5.2, first line:** Change "suppose first that $f(A) > f(B)$" to "suppose first that $f(A) < f(B)$."

(7/17/09) **Page 50, proof of Lemma 5.2, third equivalence:** The reason should be "(hypothesis and Corollary 4.15)" instead of "(definition of betweenness)."

(7/24/09) **Page 56, third line of the "Opposite Rays" section:** Change the sentence beginning "Two rays" to the following: "Two rays are said to be ***collinear rays*** if there is a line that contains both of them, and they are said to be ***opposite rays*** if they have the same endpoint and their union is a line." [Jupiter]

(8/20/09) **Page 58, paragraph just above the two figures:** "together will" should be "together with." [Cindy]

(7/24/09) **Page 67, statement of Theorem 6.3:** should stipulate that O is the endpoint of a.

(7/26/09) **Page 67, first paragraph of the "Betweenness of Rays" section, line 4:** Change $g(s)$ to $g(a)$. [Michael]

(7/27/09) **Page 70, proof of Theorem 6.9, second sentence:** Replace that sentence by the following three sentences: "Because the hypothesis implies $A \notin \overleftrightarrow{OB}$, it follows that A, O, B are noncollinear, so $\angle AOB$ is proper. Similarly, since $\overleftrightarrow{OA} = \overleftrightarrow{OC'}$, the points B, O, C' are noncollinear, so $\angle BOC'$ is proper. The angles $\angle AOB$ and BOC' share the common side $\overrightarrow{OB}$, so they do in fact form a linear pair." [Colin]

(7/26/09) **Page 71, statement of Corollary 6.14:** The first sentence should read "Suppose a, b, and c are three rays that all lie in some half-rotation, and no two of which are collinear." [Anne]

(7/27/09) **Page 72, third paragraph:** Delete the phrase "such that $\angle AOB$, $\angle BOC$, and $\angle COD$ are all proper angles. [Cindy]

(7/27/09) **Page 74, second paragraph, line 2:** after "also in the interior," insert "of $\angle AOC$."

(7/27/09) **Page 74, statement of Lemma 6.22:** Change the statement to "*Suppose $\angle AOC$ is a proper angle and $\overrightarrow{OB}$ is a ray that lies in the interior of $\angle AOC$. Then $\overrightarrow{OA} * \overrightarrow{OB} * \overrightarrow{OC}$.*" [Amy]

(7/27/09) **Page 77, statement of Theorem 6.27:** After "*in some half-rotation,*" insert "*and no two of which are collinear.*"

(7/27/09) **Page 81, proof of the crossbar theorem:** The proof should begin with the statement "Suppose $\triangle ABC$ is a triangle and $\overrightarrow{AD}$ is a ray between $\overrightarrow{AB}$ and $\overrightarrow{AC}$." [Anne]

(7/31/09) **Page 88, last line:** Change "without generality" to "without loss of generality."

(7/31/09) **Page 98, second paragraph, next-to-last line:** Insert a period after "***components***."

(7/31/09) **Page 99, equation (8.2):** Change $\mu\angle AOB$ to $m\angle AOB$. [Anne]

(8/4/09) **Page 112, Fig. 9.2:** The horizontal line should be labeled ℓ. [Anne]

(8/20/09) **Page 111, third paragraph, first line:** Delete extra "to" near the end of the line. [Amy]

(8/3/09) **Page 115, second paragraph:** Just before the last sentence of the paragraph, add "It is easy to check that all closest points in S are the same distance from P." Then in the last sentence, change the final phrase from "from P to the closest point in S" to "from P to any of the closest point(s) in S."

(8/3/09) **Page 115, statement of Theorem 9.10:** In the first sentence, add the word "proper": "*Suppose $\angle AOB$ is a proper angle ... .*"

(8/3/09) **Page 120, Problem 9.6:** Change "the supplementary angles theorem" to "the consecutive interior angles theorem."

(8/20/09) **Page 122, second line:** Change the last clause to "and no two of the segments intersect except at a common endpoint." [Cindy]

(8/5/09) **Page 124, third line from bottom:** After "correspondence between their vertices," insert: "such that consecutive vertices correspond to consecutive vertices, and ... ."

(8/20/09) **Page 134, statement of Theorem 10.15:** Remove "and only if" from each of the four parts. [Cindy, Alexei]

(8/5/09) **Page 134, Fig. 10.20:** $\angle D$ and $\angle H$ should not be marked congruent in this diagram.

(8/5/09) **Page 134, proof of Theorem 10.16, just before the last sentence:** Insert "The two triangle congruences imply $\angle ADB \cong \angle EHF$ and $\angle BDC \cong \angle FHG$, so $\angle ADC \cong \angle EHG$."

(8/5/09) **Page after #135:** This page should be numbered 136.

(8/20/09) **Page 148, second to last line:** Change $\triangle XQF$ to $\triangle XQG$. [Alexei]

(8/20/09) **Page 149, third line:** Insert missing "we": "as we will see later." [Amy]

(8/7/09) **Page 150, displayed equation near the bottom:** Change μ to m (three times).

(8/20/09) **Page 151, equation (12.1):** Change μ to m (three times). [Alexei]

(8/20/09) **Page 152, last set of displayed equations and the paragraph above them:** Change μ to m (eight times). [Alexei]

(8/20/09) **Page 153, statement of Theorem 12.18:** Change $180°$ to 180. [Michael]

(8/20/09) **Page 153, last displayed equations:** Change μ to m (six times). [Alexei]

(8/7/09) **Page 154, last line:** After "*exterior angles*," insert "(*one at each vertex*)." [Cindy]

(8/7/09) **Page 156, line above Theorem 12.30:** Change "Theorem 12.27" to "Theorem 12.28."

(8/20/09) **Page 162, third line:** Change "A and E" to "A and B": "...implies that A and B are on opposite sides of $\overleftrightarrow{EG}$; ... ." [Amy]

(8/20/09) **Page 164, last two lines before Theorem 13.7:** In these two lines, interchange the two inequalities $s(x) < x^2$ and $s(x) > x^2$. [Amy]

(8/20/09) **Page 181, Step 2, second paragraph, second line:** Change $\mathrm{HR}(A, E, C)$ to $\mathrm{HR}\big(\overrightarrow{AE}, C\big)$. [Michael]

(8/20/09) **Page 213, third line from the bottom:** Delete extra "that." [Kathryn]