Math 545Topology and Geometry of ManifoldsAutumn 2001SUGGESTIONS FOR FURTHER READING

- [1] M. Spivak, A Comprehensive Introduction to Differential Geometry, Volume 1, Publish or Perish, 1979. This is the first of a classic five-volume tome. It is a chatty and surprisingly enjoyable essay on many aspects of differential geometry. Chapters 1–3 cover much of the material we will see in Math 545, while the rest of Volume 1 contains material that will appear in 546. Its conversational style makes it very useful for getting extra insights into the basic concepts. You should sit down with a beer or a cup of tea and read this book like a novel.
- [2] Glen E. Bredon, Topology and Geometry, Springer-Verlag, New York, 1993. This beautiful book, intended as a first-year graduate text in algebraic topology, is unique among algebraic topology texts in its focus on manifolds (both the topological kind and the smooth kind) as the main objects of study throughout. Be warned that its main focus is the topology, so the smooth manifold theory is given rather short shrift. Most of the material in Chapter II, Sections 1–7 and 9–11, will be covered in Math 545, while Sections 8 and 12 will be covered in Math 546.
- [3] F. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer-Verlag, 1983. For many years this was the classic reference for introductory smooth manifold theory. Most of the material in Math 545 is covered in the first 35 pages, so beginners generally find it too dense and formal to be a good text. But a whole generation of mathematicians have been brought up on its treatments of Lie groups, de Rham theory, and Hodge theory.
- [4] W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Second Edition, Academic Press, 1986. Covers most of the material in Math 545 and 546, although with some notable omissions (e.g., the Whitney approximation and embedding theorems and the global Frobenius theorem) and some odd choices of ordering (e.g., the rank of a smooth map is defined before the tangent space). Fairly easy to read, and good for an alternative perspective.
- [5] S. Sternberg, Lectures on Differential Geometry, Prentice-Hall, 1964. Beginning with a treatment of linear algebra and smooth manifold theory, this text moves rapidly into some very advanced topics, including Sard's theorem, bundle theory, Riemannian geometry, Lie groups, homogeneous spaces, and G-structures. Can be rewarding, but not for the faint-hearted. The Math 545 material is mostly found in Chapter II, Sections 1-5.
- [6] S. Lang, *Fundamentals of Differential Geometry*, Springer–Verlag, 1999. As in all of Lang's books, everything is done in the utmost generality and with great economy.

Its most notable and useful feature is that it treats infinite-dimensional manifolds on an equal footing with finite-dimensional ones. Chapters I–III correspond roughly to Math 545. Much of the same material can also be found in the earlier incarnations of this book: *Introduction to Differentiable Manifolds*, Interscience, 1962; *Differential Manifolds*, Addison–Wesley, 1972, or Springer-Verlag, 1985.

- [7] T. Aubin, A Course in Differential Geometry, AMS, 2000. This one is rather terse and suffers from the writer's unfamiliarity with idiomatic English, but it's useful for getting an idea of a modern geometric analyst's perspective on smooth manifold theory. Most of the Math 545 material is in Chapter 1 and the first half of Chapter 2.
- [8] M. W. Hirsch, Differential Topology, Springer-Verlag, 1976. This nice little book focuses on properties of smooth manifolds that are preserved by diffeomorphisms, and their applications to topology and to the classification of manifolds up to diffeomorphism. Its main tools are approximation and embedding theorems, Sard's theorem and transversality, Morse theory, and surgery.
- [9] S. S. Chern, Complex Manifolds Without Potential Theory, Second Edition, Springer– Verlag, 1979. A good introduction to the theory of complex manifolds, a subject that is far deeper than just smooth manifold theory with the word "smooth" replaced by "complex-analytic." Chapters 1–3 cover material that roughly corresponds (in the complex setting) to Chapters 1–4 of [ISM]. (The odd title refers to the fact that the book does not treat Hodge theory—the application of partial differential equations to the study of differential forms—which is a standard tool in most introductory complex manifold texts.)
- [10] R. O. Wells, Jr., Differential Analysis on Complex Manifolds, Springer-Verlag, 1980. Chapter I contains another good introduction to complex manifold theory. (The main goal of this book, in contrast to [9], is to develop Hodge theory for complex manifolds and its applications. Thus it could be called "complex manifolds with potential theory.")