Math 548 **Geometric Structures** **Winter 2019**

SYLLABUS

Lectures: MWF 1:30–2:20
Padelford C-401

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Course Web site: *www.math.washington.edu/~lee/Courses/548-2019*

Text: The only required text is the notes that I will be writing. They'll be posted in PDF format on the class website as they become available.

Prerequisites:

Prerequisite: Math 547 (Riemannian Geometry) or equivalent. Math 564 (first quarter of Algebraic Topology) would be helpful, but is not required.

General description:

Vector bundles and fiber bundles appear everywhere in differential geometry, and the techniques for analyzing them are central to modern differential geometry and its applications. This course will focus primarily on the differential-geometric aspects of bundles, with the aim of developing both intuition and facility with the technical machinery.

Main Topics:

- Vector bundles and fiber bundles
- Whitney sums, tensor products, pullback bundles
- Structure groups, G-structures, and reduction of structure groups
- Principal bundles and associated bundles
- Classifying maps and universal bundles
- General definition of characteristic classes
- Connections on vector bundles and principal bundles
- Curvature of arbitrary connections, Bianchi identities
- Flat connections and representations of π_1
- Chern-Weil theory and the Chern–Gauss–Bonnet theorem

That's probably already too much for one quarter; but if there's time, I might touch on some or all of the following:

- Spin structures, spin bundles, and spinors
- Sheaves, sheaf cohomology, and the generalized de Rham theorem
- Gauge theories

Homework:

Problem sets will be assigned at irregular intervals, usually every two weeks or so. When you write them up, please follow the following guidelines:

- **Collaboration:** I strongly encourage you to work with other students on the homework. You'll get the most benefit from working with others if you make a good faith effort to solve the problems on your own first; but once you've thought about them for a while, I don't mind if you ask each other for explanations of how to do the problems. When writing up solutions to hand in, *you must write your own solutions in your own words.*
- **Citing results:** You may freely cite theorems, lemmas, propositions, and exercises from the notes. You may also use results from my books *Introduction to Topological Manifolds*, *Introduction to Smooth Manifolds*, and *Introduction to Riemannian Manifolds*, including problems and exercises, unless they are essentially what you're being asked to prove. If you do, be sure to state clearly what result you're using, either by giving a theorem number and saying which book and which edition it comes from, or by the name of the theorem, or by stating the theorem briefly. Please don't consult any other sources, including other books or the Internet.
- **Problem Statements:** You need not copy the entire problem statement, but be sure to state clearly what you're proving. I prefer that you state each result in the form of a theorem (e.g., "**Theorem:** Every flabby sheaf is soft") instead of a command ("Prove that every flabby sheaf is soft") or a question ("Is every flabby sheaf soft?").
- **Assembly:** Arrange your solutions in numerical order, just as they appear on the assignment page, with each problem starting on a new page. Problems that are out of order might not get credit. Please staple the pages of each assignment together.
- **Identification:** Make sure the first page of each homework packet is clearly labeled with your name and the assignment number.
- **Typesetting vs. handwriting:** I strongly encourage you to submit computer-typeset assignments. I recommend LaTeX, since it's the de facto standard in mathematics, and you'll have to learn it sooner or later if you continue doing math research or teaching; but any typesetting program will do. I've posted some helpful typesetting links on the class web page. I'm also happy to accept handwritten assignments, as long as they are neat and legible (see below).
- **Legibility:** If you write by hand, write your answers neatly and legibly, not too small, with as few erasures or crossouts as possible. Be sure to distinguish clearly between similar symbols, such as a/α, $b/6$, $C/\subseteq$, $\in/\varepsilon$, $g/q/9$, h/n, $I/l/1$, p/ρ, r/γ, $s/5$, $t/+$, $u/v/\nu$, $U/\cup$, $x/\times/\chi$, $y/4$, $z/2$, ζ/ξ, and uppercase/lowercase letters. Unless mathematical ideas spring fully and impeccably realized from your pen, your first draft is not acceptable.
- **White space:** Don't be stingy with white space. *Leave one-inch margins on all four sides of your pages.* If you don't, I'll be annoyed because I don't have room to write comments, and you don't want your paper being graded by an annoyed reader!

Grading:

Your grade will be based on homework; there are no exams. Roughly speaking, the cutoff for a 4.0 will be somewhere around 80%, and the cutoff for a 3.0 somewhere around 50%.

If you wish, you may register for this course on an S/NS basis (for example, if you have passed prelims and chosen a PhD committee, and are therefore no longer required to register for graded courses). In this case, if you attend regularly and hand in at least two correct written problem solutions, I will record your grade as a 2.7, which will be converted by the registrar to S (satisfactory).