

PROBLEMS DISCUSSED AT THE SPRING 2006 PNGS

1. PROBLEMS ON THE HAMILTONIAN HOMEOMORPHISM GROUP, BY YONG-GEUN OH

- (1) Describe the closed set of length minimizing paths in terms of the geometry and dynamics of the Hamiltonian flows.
- (2) Describe the set of topological Hamiltonians in $L_m^{(1,\infty)}([0, 1] \times M, \mathbb{R})$.
- (3) Study the structure of the flow of Hamiltonian homeomorphisms in terms of the C^0 -Hamiltonian dynamical system or as the high dimensional generalization of area-preserving homeomorphisms with vanishing mass flow or zero mean rotation vector.
- (4) Does the identity $[Sympeo_0, Sympeo_0] = Hameo$ hold? Is $Hameo$ simple?
- (5) Is $Hameo(M, \omega)$ a Lie group?

2. PROBLEMS ON DONALDSON-THOMAS AND GROMOV-WITTEN INVARIANTS OF ORBIFOLDS AND THEIR CREPANT RESOLUTIONS, BY JIM BRYAN

- (1) Define Donaldson-Thomas invariants for orbifolds and formulate (1) an orbifold version of the Gromom-Witten/Donaldson-Thomas correspondence and (2) the Crepant Resolution Conjecture for Donaldson-Thomas invariants. The GW/DT correspondence (for non-orbifolds) was conjectured by Maulik-Nekrasov-Okounkov-Pandharipande and states that the reduced partition functions of the Donaldson-Thomas and Gromov-Witten invariants are equal after a certain change of variables. The Crepant Resolution Conjecture for Gromov-Witten theory has been formulated by Bryan-Graber and states that the Gromov-Witten partition functions of a Gorenstein orbifold and any crepant resolution are equal after a certain change of variables.
- (2) Extend the definition of Donaldson-Thomas invariants to the symplectic category (they are currently only defined for algebraic manifolds). Thomas's original motivation and the conjectural correspondence with Gromov-Witten theory both suggest that Donaldson-Thomas theory should be defined in the symplectic category.
- (3) Find a direct geometric characterization of the Gromov-Witten potential. In physics, the Gromov-Witten potential has intrinsic meaning but in mathematics, it is currently only formally defined – only the coefficients of the formal series have a geometric meaning.

3. PROBLEMS ON GLOBAL COVERGENCE OF THE YAMABE FLOW, BY SIMON BRENDLE

- (1) Does the positive mass theorem hold in all dimensions and for all topologies? An affirmative answer to this question would imply the convergence of the Yamabe flow on any manifold.
- (2) It would be interesting to gain a better understanding of the blow-up issues for the prescribed scalar curvature equation.

4. PROBLEMS ON THE KÄHLER-RICCI FLOW AND MONGE-AMPERE EQUATION, BY
GANG TIAN

Let (X, ω) be a Kähler manifold. Consider the normalized Kähler-Ricci flow

$$\begin{aligned} \frac{\partial \omega}{\partial t} &= -(\text{Ric}(\omega) - \lambda \omega) \\ \omega(0, \cdot) &= \omega_0 \quad \lambda = -1, 0, 1 \end{aligned}$$

- (1) ($\lambda = 1$) Conjecture: There exists $\varphi_t : X \rightarrow X$ such that $\varphi_t^* \omega(t, \cdot)$ converges in C^∞ to a Kähler-Ricci soliton (possibly on a different “manifold”).
- (2) By results of Tian-Zhang, Carsini-LaNave, Zhang, there exists solution $\omega(t, \cdot)$ for $t \in [0, T)$ where $T = \sup_t \{[\omega_t] > 0\}$, $\omega(t, \cdot) \rightarrow \omega(T, \cdot)$ (current) as $t \rightarrow T$ s.t. $\omega(T, \cdot)$ is smooth on $X - D$ and locally, $\omega(T, \cdot) = \partial \bar{\partial} \varphi$ for continuous φ . Speculations: Take X_T to be the metric completion of $X - D$ with respect to $\omega(T, \cdot)$. Then φ descends to X_T . Questions: 1) Does X_T exist as a variety? 2) How to define Ricci flow on X_T with only continuous initial data? 3) Why are there only finitely many T ? 4) What is the limit metric at $t = \infty$. Some partial answers discussed.

5. PROBLEMS ON SEMI-DISCRETE CURVATURE FLOWS, BY BEN CHOW

- (1) Combinatorial Yamabe flow $\frac{d}{dt} r_i = -K_i r_i$. When does this converge? Long time existence?