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Collapsing Sequences of Constant Mean Curvature Surfaces in Riemannian Manifolds

Some open problems

1. Using the gluing method for constructing CMC surfaces of high mean curvature out of small spheres, is it possible to construct very "interesting" surfaces such as bent surfaces?

2. In the scenario above, how do the properties of the Riemann curvature tensor at a point p in the manifold impinge upon the types of CMC surfaces we can construct near p using this method?

3. Suppose we construct extended surfaces of this type, built by positioning spheres end-to-end along a curve γ. Except this time, we no longer make all the symmetry assumptions of the Butscher/Mazzeo theorem. What type of curve is γ? A geodesic or a flow line of the scalar curvature or something else?

4. Suppose Σ_r is a sequence of CMC surfaces in M having mean curvature $2/r$. Suppose that M_r "collapses" onto a one- or zero- dimensional variety Γ in the following sense: for every r, the surface Σ_r is contained in a tubular neighbourhood of Γ of radius $10r$. What can we say about Γ? What can we say about Σ_r as r is very small? What can we say about the limit of the sequence of surfaces?