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The Affine Bonnet Problem

The classical Euclidean problem studied by Bonnet was to determine whether, and in how many ways, a Riemannian surface can be isometrically embedded into Euclidean 3-space so that its mean curvature is a prescribed function. He found that, generically, specifying the metric and mean curvature allowed no solution but that there are special cases in which, not only are there solutions, but there are even 1-parameter families of distinct solutions. Much later, these 'Bonnet surfaces' were found to be intimately connected with integrable systems and Lax pairs.

In this talk, I will consider the analogous problem in affine geometry: To determine whether, and in how many ways, a surface endowed with a Riemannian metric g and a function H can be immersed into affine 3-space in such a way that the induced Blaschke metric is g and the induced affine mean curvature is H. This affine problem is, in many ways, richer and more interesting than the corresponding Euclidean problem. I will classify the pairs (g, H) that display the greatest flexibility in their solution space and tell what is known about the (suspected) links with integrable systems and Lax pairs.