

NAME: _____

Student ID #: _____

QUIZ SECTION: _____

Math 111 D
Midterm I -- Solutions
October 19th, 2006

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Problem 4	20	
Total:	50	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 4 pages in total and 4 problems. Please check that your test is complete!
- You **must explain how you get your answers**. Correct (or incorrect) answers with no supporting work may result in little or no credit. On problems in which you use a graph, draw lines, label them, and mark points clearly.
- Write your **final answer in the indicated spaces**. Unless otherwise noted, round your answer to two decimal digits.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

GOOD LUCK!

Do you want me to post your grade so far next week on the class website under the last 4 digits of your student ID?

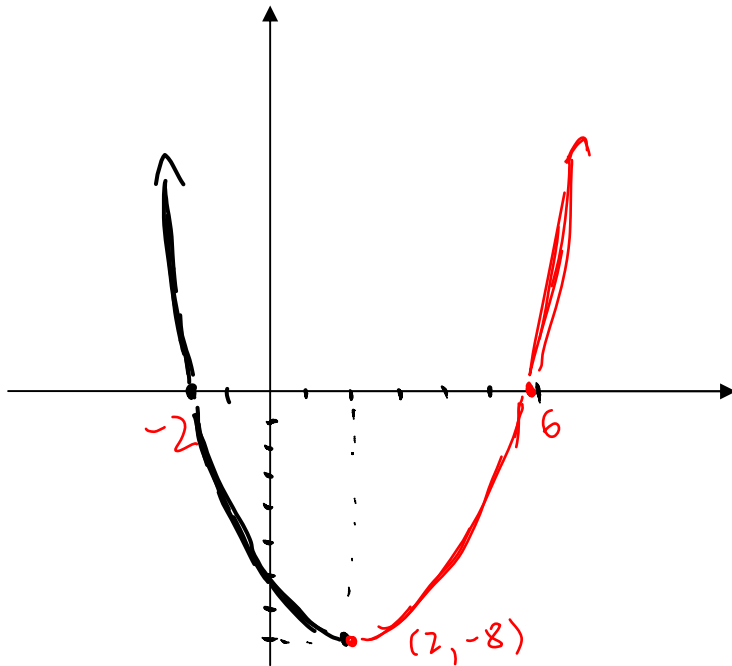
Yes, please post my grade. Sign to give permission: _____

No, please don't post my grade so far.

1 (6 points) Graph the following quadratic function: $f(x) = \frac{1}{2}x^2 - 2x - 6$.

Label both coordinates of its vertex. Also, label its intersection points with the x-axis.

(Show work and indicate which formulas you used – using a graphing calculator will not get much credit)



Shape: concave up parabola

Vertex:

x-coordinate via vertex formula:

$$\frac{-b}{2a} = \frac{-(-2)}{2\left(\frac{1}{2}\right)} = \frac{2}{1} = 2$$

& y-coord: $f(2) = -8$

x-intercepts via quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4(1/2)(-6)}}{2(1/2)}$$

$$= \frac{2 \pm \sqrt{16}}{1} = -2 \text{ \& } 6$$

2 (10 points: 3+3+4) The following table indicates the (incremental) average rate of water usage from a reservoir, over successive 5 minute time intervals.

Time interval (in min.)	0 → 5	5 → 10	10 → 15	15 → 20	20 → 25	25 → 30	30 → 35	35 → 40	40 → 45
Rate of Usage (in gallons per min)	3.5	1.7	2	2.8	3.3	4.4	10	2.9	5

a) How much water was used during the first 5 minutes?

Work: $(3.5 \text{ gal/min}) \times (5 \text{ min}) = 17.5 \text{ gal}$

Answer: 17.5 gallons.

b) How much water was used from $t=15$ to $t=30$ minutes?

Work: compute the amount used over each 5 minute interval, then add up the amounts

$$2.8 \times 5 + 3.3 \times 5 + 4.4 \times 5 = 52.5$$

Note: It is not OK to add up the rates first, then multiply by 15 minutes! You'd get a different (and incorrect answer)

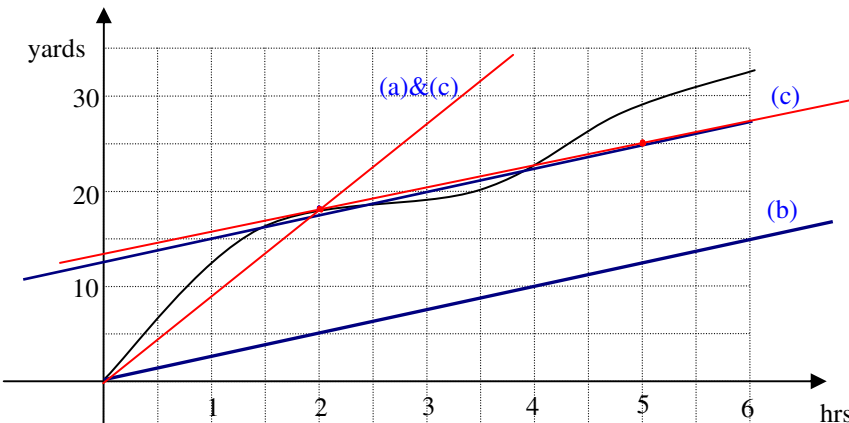
Answer: 52.5 gallons.

c) What was the overall average rate of water usage, over the entire 45 minutes?

$$\text{Work: total water / total time} = \frac{3.5 \times 5 + 1.7 \times 5 + \dots + 5 \times 5}{45} = \frac{178}{45} = 3.96$$

Answer: 3.96 gallons per minute.

3 (14 points: 4+4+6) The graph below represents the distance $D(t)$ (in yards) traveled by the Mars Rover vehicle at time t (in hours), starting at noon.



NOTE: In all questions below we accepted a range of values as correct, as long as correct work was shown and the lines/measurements used were not too far off.

a) What was the average trip speed of the Rover two hours into the trip? (include units)

*Draw a diagonal line to the point on the graph above $t=2$. Measure the slope.
OR: use $ATS(2)=D(2)/2=18/2=9$*

Answer: 9 Units: yards per hour

b) Find a one hour time interval over which the Rover traveled 2.5 yards.

Draw a reference line of slope 2.5 (diagonal through point (2,5)). Keep your ruler parallel to this and roll it until it intersects the graph at two points which are 1 hr (2 horizontal tickmarks) apart. There are two possible answers: about 1.4 to 2.4 and about 2.7 to 3.7. We accepted a range around these values.

Answer: from $t = \underline{1.4}$ to $t = \underline{2.4}$ hours.

c) Consider the following statement in functional notation: $\frac{D(2+3) - D(2)}{3} > \frac{D(2)}{2}$.

i) Translate it into English:

Answer:

The car's average speed over a 3-hour interval starting at $t=2$ (or: from 2hrs to 5 hrs) was greater than the car's average trip speed after 2 hrs.

ii) Translate it into graph language:

Answer:

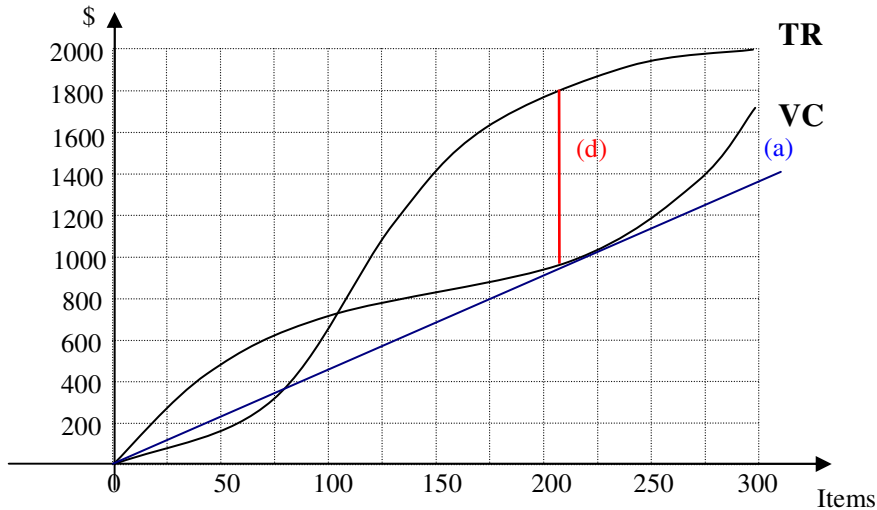
The slope of the secant line through the distance graph at $t=2$ and $t=5$ is greater than the slope of the diagonal line through the distance graph at $t=2$.

iii) Use the graph to determine if the statement true or false.

See the red lines

Answer (circle one): True or False

4 (20 points) The following are the graphs of the the total revenue (TR) and the variable cost (VC), in dollars, for producing and selling Items.



a) For this VC curve, what is the shutdown price in a market price situation?

Work:

*Draw the lowest slope diagonal line which intersects the graph of VC. It passes thru (175, 800).
SDP=Slope of this line= $800/175=4.57$*

Answer: SDP = 4.57 \$ per Item.

b) What is the Average Revenue from selling 50 Items?

Work:

*$AR(50)=TR(50)/50 \approx 175/50=3.5$
OR: slope of diag line to TR at $q=50$.*

Answer: AR(50) = 3.5 \$ per Item.

c) Give an interval of quantities q over which the marginal revenue MR is decreasing.

Work:

*Looking for a range of quantities so that the tangents to TR become less steep with larger q .
This happens for all q larger than about 125, so pick an interval above that value..*

Answer: from $q =$ 150 Items to $q =$ 300 Items

d) Suppose the fixed cost is \$250. Find the maximum profit, and at what quantity q it occurs.

Work:

*Keep ruler vertical and roll it looking for the largest vertical distance between TR (top) and VC (bottom).
This occurs at about $q=210$ items (range: 205 to 245).
The distance is about 4 vertical tickmarks, so about \$800.
So the max $TR-VC=\$800$
Profit= $TR-TC=TR-VC-FC$. Max profit is $\$800-\$250=\$550$*

Answer: Maximum profit is \$ 550 for $q =$ 210 Items