

SOLUTIONS

Nichifor, AUT 2007, Midterm 1, version 1

1 (4 points) Find both coordinates of the vertex for the graph of the following quadratic function:

$$y = f(x) = -5x^2 - 8x + 17$$

Use the Vertex Formula to get the x—coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{-8}{2(-5)} = -0.8$$

Evaluate the function at $x = -0.8$ to get the y-coordinate:

$$y = f(-0.8) = -5(-0.8)^2 - 8(-0.8) + 17 = 20.2$$

Answer: $x = \underline{-0.8}$, $y = \underline{20.2}$

2 (8 points) Let $D(t)$ represent the distance traveled by a bicycle (in **kilometers**), up to time t (in **hours**), starting from the bike's initial position (that is, $D(0)=0$).

a) Translate the following statement into **English** (including the appropriate units):

$$\frac{D(2)}{2} = 18.$$

Answer: "The average trip speed of the bike at 2 hours was 18 kilometers per hour."

b) Translate the following statement into **functional notation**:

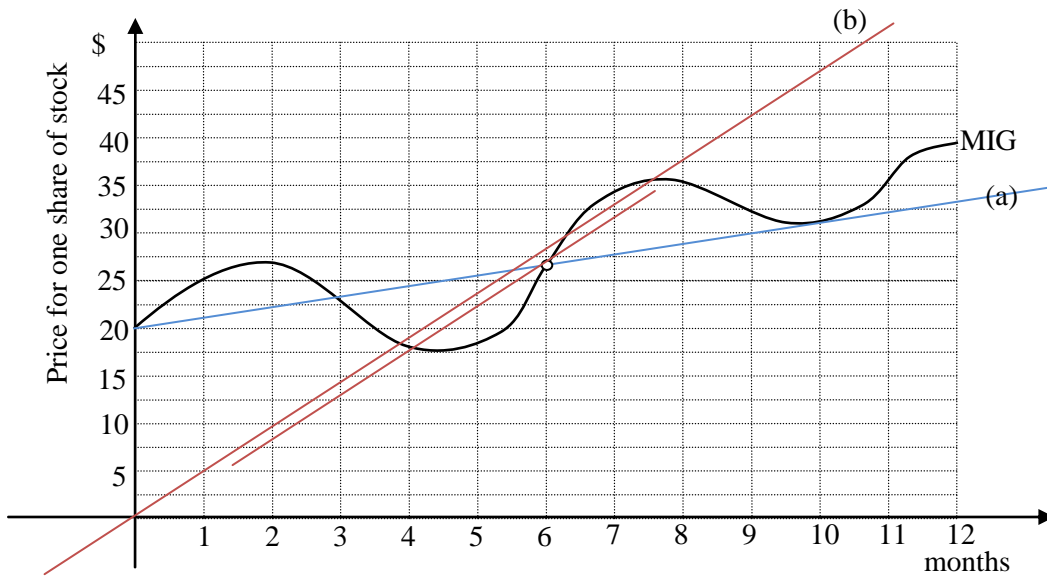
"The bike traveled 15 kilometers from 1 hour to 2 hours"

Answer:

$$D(2) - D(1) = 15$$

3 (10 points)

The graph below shows the stock price (in \$) for one share of a company named MathIsGreat (MIG), over the course of 12 months. Note that the initial price for one share of this stock was \$20.



- a) What was the average rate of change of the stock price over the first 6 months? (include correct units)
*(*note: this was like the problem on Activity 1. In particular, you had to subtract the initial value of the stock)*

Method 1: $\frac{\Delta \text{MIG}}{\Delta t} = \frac{\text{MIG}(6) - \text{MIG}(0)}{6} \approx \frac{\$27.5 - \$20}{6 \text{ months}} \approx 1.08 \text{ \$ per month}$

Method 2: slope of secant from $t=0$ to $t=6$ (Since the graph does not start at origin, you cannot use the diagonal)
 Draw line (a), pick points (0, 20) and (7, 27.5).
 Slope = $(\$27.5 - \$20) / 7 \text{ months} = 1.07 \text{ \$ per month}$

Answer: Rate: ≈ 1.07 Units: \$ per month

- b) Find a 2-month time interval, if any exists, over which the price of one share of MIG increased by \$10.

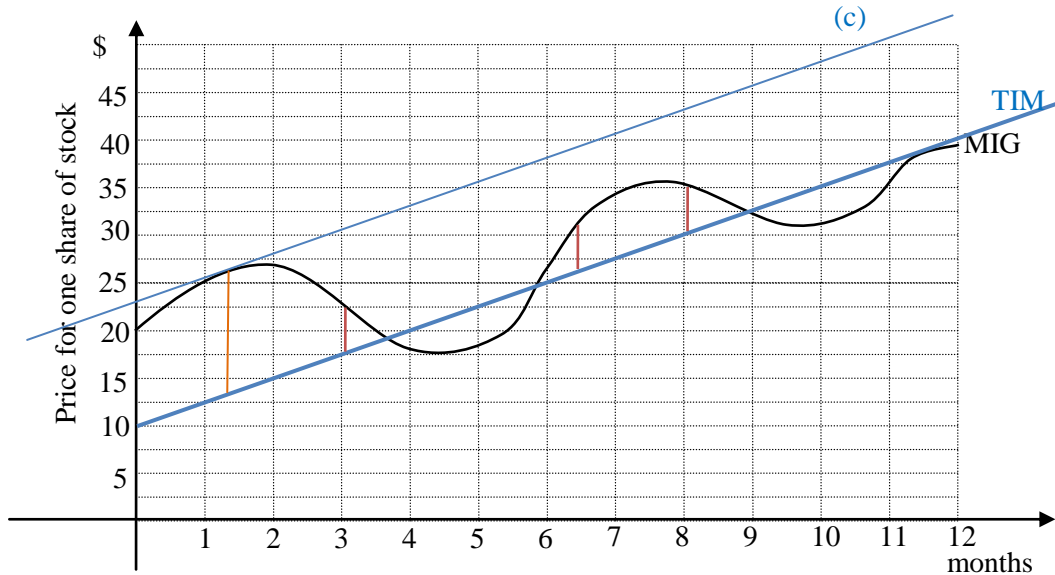
We're looking for 2 points, 2 months apart, where the slope of the secant through these points is \$10/2 months, i.e. 5 \$ per month.

Draw a reference diagonal line (b) of slope 5.
 Move ruler parallel to this line until it intersects the graph at 2 points two months apart.
 There were 2 correct answers: from about 4.1 to about 6.1, or from about 5.9 to 7.9.

Answer: from $t=$ _____ to $t=$ _____ months
(these times need not be whole numbers of months)

4 (12 points)

The following is the same graph as in the previous problem, and recall that it shows the stock price (in \$) for one share of a company named MathIsGreat (MIG), over the course of 12 months.



- a) Suppose another stock, TimeIsMoney (TIM), starts off at \$10 per share at $t=0$, and its share price increases by \$2.50 every month. On the axes above, draw the graph for TIM's share price for the 12-month period.

Draw a line with y-intercept 10, and slope 2.5.

- b) Find **all times** t when MIG's share price is \$5 higher than TIM's.

Find times when the vertical distance between the two graphs is \$5 (2 vertical tick marks), with MIG above TIM. (see red vertical segments in picture)

Answer: At $t = \underline{3.1, 6.4, 8.1}$ months (list all).
(answers need not be a whole number of months)

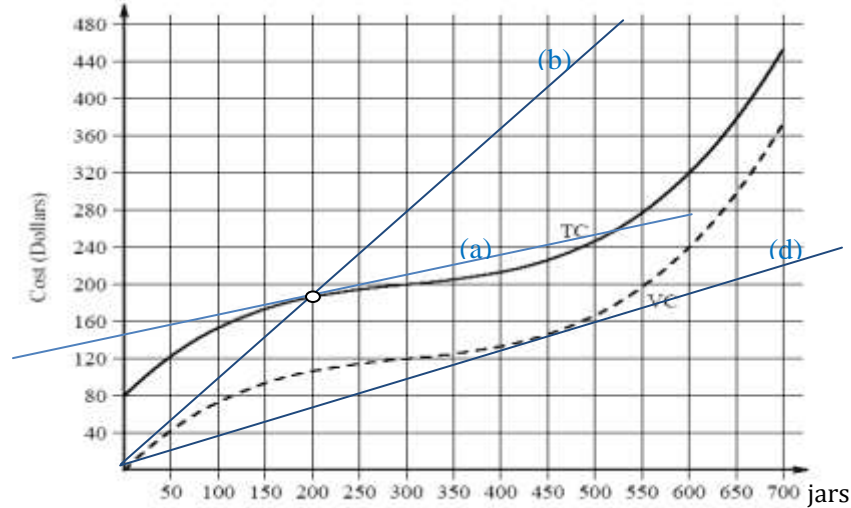
- c) Find the greatest price difference between the share prices of the two stocks during the 12-month period.

Best method: Place ruler parallel to TIM, roll it parallel until it becomes tangent to MIG at the point corresponding to largest difference (see line (c)). Then measure that difference.

Answer: \$ 13.

SOLUTIONS

5 (16 points) The graphs below represent the Total Cost (TC), and the Variable Cost (VC), for the Hungry Baby Company, which is producing and selling jars of baby food.



a) Compute the Marginal Cost for producing the 201st jar.

Draw the tangent line to TC at 200 and compute its slope:
pick pts (250, 200) and (450, 240) on the tangent. Slope=(240-200)/(450-250)=40/200=0.2

Answer: MC(200)= 0.2 dollars.

b) Compute the Average Cost for producing 200 jars. Include correct units.

Draw the diagonal line (b) through the point on TC at q=200 and compute its slope=320/350=0.91
OR:

Use the formula and read from graph: AC(200)=TC(200)/200=185/200=0.925

Answer: AC(200)= 0.91 Units: \$ per jar .

c) Find a range of quantities, if any exist, over which the Marginal Cost is increasing, but the Average Cost is decreasing.

Use appropriate rolling ruler methods to find an interval over which:

- 1) the slopes (MC) of the tangent lines to TC at successive points are increasing ($q > 300$),
- 2) AND the slopes (AC) of the diagonal lines through successive points on TC are decreasing ($q < 500$).

So any interval between about 300 and 500 would work.

Answer: From $q =$ 350 to $q =$ 450 jars.

d) Find the shutdown price.

Draw the lowest diagonal line that's tangent to VC (line (d) on graph) and compute its slope:

Say, use the pt (250, 80): slope =80/250=0.32

Answer: SDP = 0.32 \$ per jar.