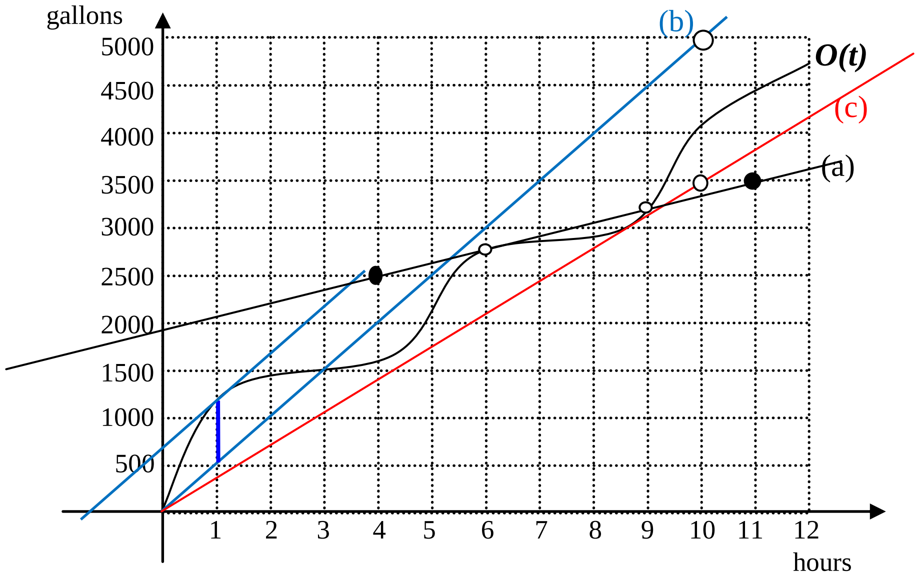
version 1

1. **(15 points)** The graph below is of the total amount of the water ***O(t)*** which was pumped **out** of a reservoir from noon until *t* hours later (during a 12 hour interval, starting at noon).



a) Compute the average rate of water pumped out of the reservoir from $t = 6$ hours to $t = 9$ hours.

Draw the secant line through the points at t=6 and t=9, pick two far apart, easy to read points on it, and compute its slope:

$$\frac{3500 - 2500}{11 - 4} = \frac{1000}{7} \approx 142.8$$

ANSWER: ______142.8__________ gallons per hour

(We accepted a reasonable range of values, as long as correct work was shown and the estimates were not too far off.)

b) A pipe brings water **into** the reservoir at the constant rate of 500 gallons per hour. What is the lowest initial amount of water needed in the reservoir, in order not to run out at any time before midnight?

$$Amount\ needed = max\ deficit = max\ [O(t) - I(t)]$$

Since the inflow rate is constant at 500 gal/hr, the total water flowing in, I(t), is a diagonal line of slope 500.
Draw I(t). We're only interested in the portion where O(t) is above I(t).
Roll the ruler parallel to I(t) until it becomes tangent to O(t). Read max gap at that point.

ANSWER: We need at least ___700____ gallons of water in the reservoir initially

c) Find the lowest overall average rate of water pumped out of the reservoir during this 12 hour time interval.

Overall average rates of water drawn out of the reservoir correspond to slopes of diagonal lines to O(t).
Draw the lowest diagonal line that touches O(t) and measure its slope =3500/10=350

Answer: ____350_____ gallons per hour.

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2. **(8 points)** Let $P(t)$ denote the outside temperature, in degrees Fahrenheit (°F), at t hours past midnight.

a) Translate the following statement into a complete **English** sentence (including the correct units):

$$\frac{P(12) - P(6)}{12 - 6} = 10$$

Translation:

The (incremental) average rate of change of temperature from 6am to noon is 10°F per hour.

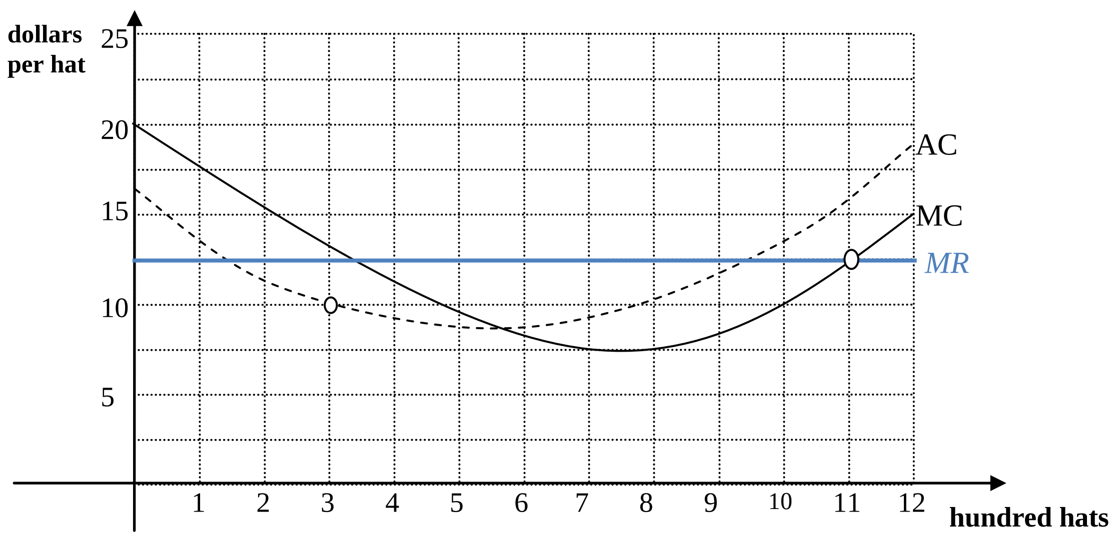
b) Translate the following statement into **functional notation:**

"The temperature decreased by 5°F from 3am to 5am."

Translation:

$P(5) - P(3) = -5$

3. (**10 points**) You run a business producing Halloween costumes. The graphs below represent the marginal cost (MC) and the average cost (AC), in dollars per item, for producing Dumbledore Wizard Hats. Note that the quantity q is in hundreds of hats.



a) If every Dumbledore Hat sells at a market price of \$12.50 per hat, what quantity maximizes your profit?
Since all you have is the graph of Marginal Cost (not Total Cost!), use method 3 of maximizing profit: Draw MR=\$12.50 and see where it crosses MC.
Look for the crossing point where MR switched from being above MC before to being below MC after. Looking at the graphs, this is the 2nd intersection point.

ANSWER: q = ________11_______ hundred hats

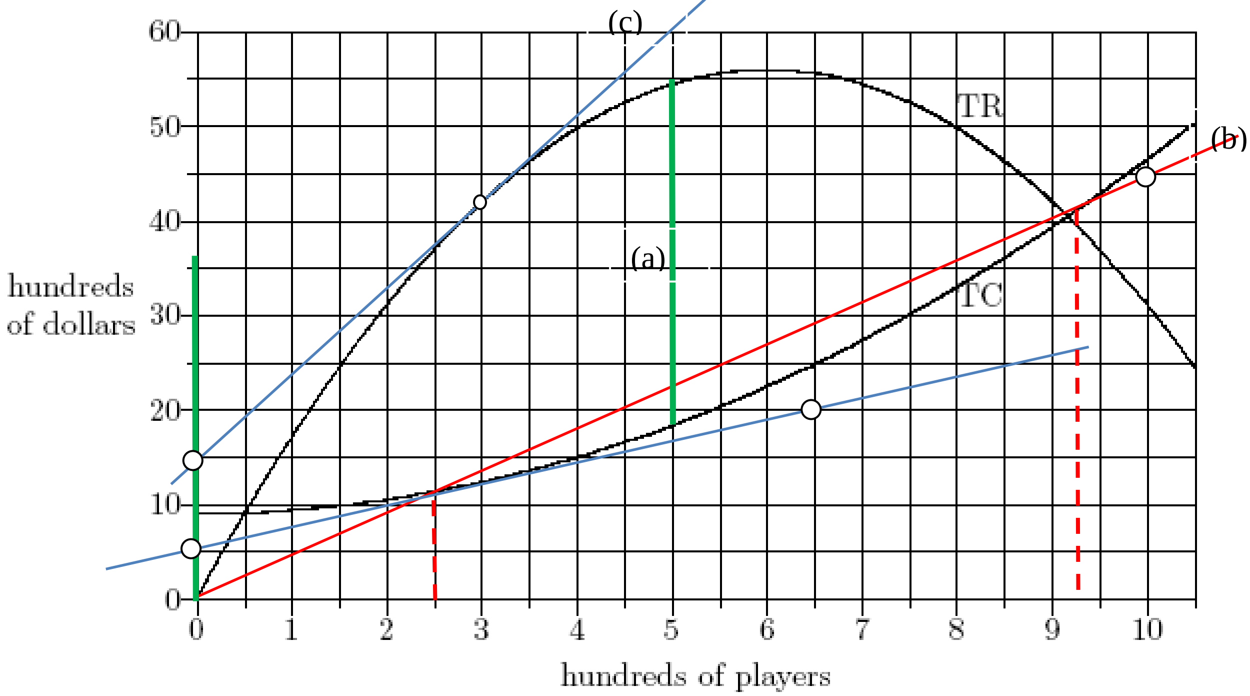
b) What is the total cost of producing 3 hundred hats? Give your answer in dollars.

$$TC(3) = AC(3) \times 3 = \left(10\ \frac{\$}{\cancel{hat}}\right) \times (3\ hundred\ \cancel{hats}) = 30\ hundred\ dollars$$

ANSWER: $TC(3)$ = \$_________3,000___________________

version 1

4. **(17 points)** You own a business producing MP3 players. The graphs below represent the total cost (TC) and total revenue (TR), in hundreds of dollars. Note that the quantity q is in hundreds of players.



a) Carefully estimate your maximal profit.

Estimate the max vertical distance between TR and TC with TR above TC. Accepted a reasonable range of values near 36 hundred \$, if correct work was shown.

ANSWER: Max profit : ___36____ hundred dollars

b) At what quantity is the average cost (AC) \$4.5 per player?

Draw a diagonal line of slope 4.5, and find the q where it crosses the TC graph.

ANSWER: at q = ___2.5 (or 9.3)_____ hundred players

c) Compute both the **marginal revenue** and the **marginal cost** at 3 hundred players. Include correct units.

Since the quantities are in hundreds, we cannot read points 1 unit apart on these graphs. So, draw tangent lines to TR and TC at q=3, pick good points on them and measure the slopes. We accepted a reasonable range of values, provided correct work was shown.

$$MR(3) = \frac{42 - 15}{3 - 0} = \frac{27}{3} = 9$$

$$MC(3) = \frac{20 - 5}{6.5 - 0} = \frac{15}{6.5} \approx 2.3$$

ANSWER: $MR(3)$ =_____9______, $MC(3)$ =_____2.3_____, Units:____\$ (per player)_______

(Accepted units:" \$"," \$ per player", or "hundreds of \$ per hundreds of players", though the last one should' been simplified to "\$ per player")

d) Will your **profit** increase or decrease if you produce and sell 301 players instead of 300 players? By how much?

$$\Delta Profit = MR(3) - MC(3) = 9 - 2.3 = 6.7$$

Note: Since q is in hundreds, TR(4) is TR at 400 players , not at 301, so it's incorrect to take MR(3)=TR(4)-TR(3)!

ANSWER: Profit will increase/decrease (circle the correct one) by \$ ___6.7____