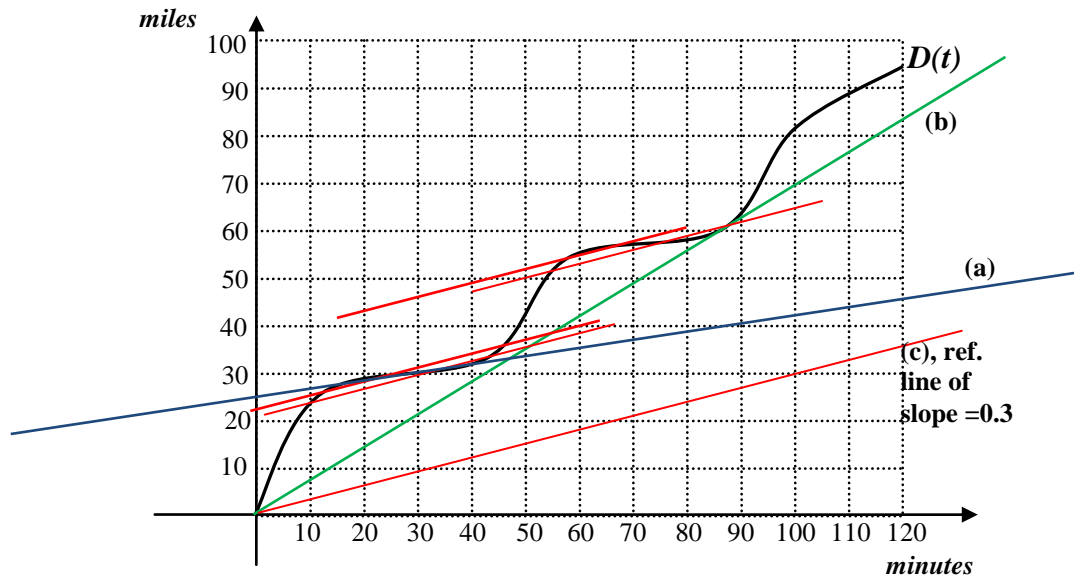


1 (12 pts) The following is the graph of the distance  $D(t)$  traveled by a car at time  $t$  minutes, during a two hour trip.



a) Compute the car's average speed from 20 to 40 minutes. Show work.

Draw the secant line through  $t=20$  and  $t=40$ . Slope of of line (a)  $\cong \frac{40-30}{90-30} = \frac{10}{60} \cong 0.17$

(range: [0.15, 0.23])

ANSWER: 0.17 miles per minute

b) What was the car's lowest average trip speed? Show work.

Lowest ATS= slope of lowest diagonal line thru  $D(t)$  = slope of line (b)  $\cong \frac{70}{100} = 0.7$

(range: [0.67, 0.72])

ANSWER: lowest ATS = 0.7 miles per minute

c) Draw an appropriate reference line and use it to find two different times  $t$  at which the following equality is true:

$$\frac{D(t+10) - D(t)}{10} = 0.3$$

*Graph translation: the slope of the secant line from  $t$  to  $t+10$  is 0.3*

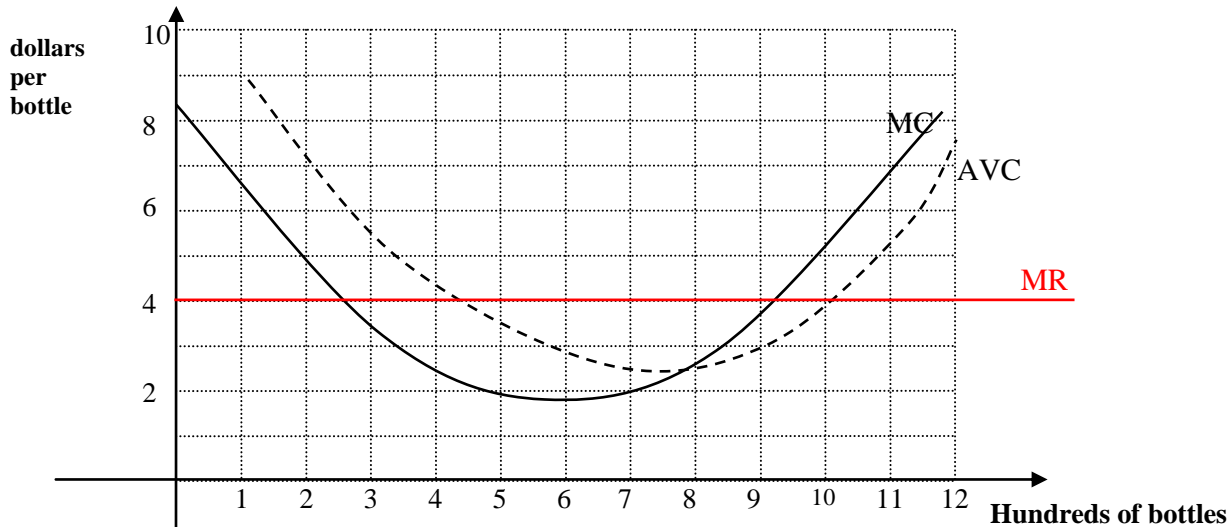
*Draw a reference line of slope  $0.3=3/10=30/100$ , as pictured.*

*Keep ruler parallel to this reference line and move up until it crosses the graph at two points that are 10 minutes apart.*

*There are 4 possible answers: 13 to 23 minutes, 30 to 40 minutes, 58 to 68 minutes, or 75 to 85 minutes. Pick the starting times of any two of these. For instance:*

ANSWER: at  $t =$  13 and at  $t =$  75 minutes

**2** (10 points) You produce and sell bottles of Love Potion #9. The graphs below represent your marginal cost (MC) and the average variable cost (AVC), in dollars per bottle. Note that the quantity  $q$  is in hundreds of bottles.



a) (3pts) What is the shutdown price?

SDP = y-coordinate of lowest point of AVC = y-coordinate of point where MC crosses AVC  $\approx 2.5$   
 (range: [2.3, 2.5])

ANSWER: SDP = 2.5 dollars per bottle

b) (3pts) If every bottle sells at a market price of \$4 per bottle, what quantity maximizes your profit?

Draw MR: horizontal line at height 4.  
 MR=MC (switching from MR above MC to MR below MC) at  $q=9.2$  hundred bottles.  
 (range: [9.1, 9.3])

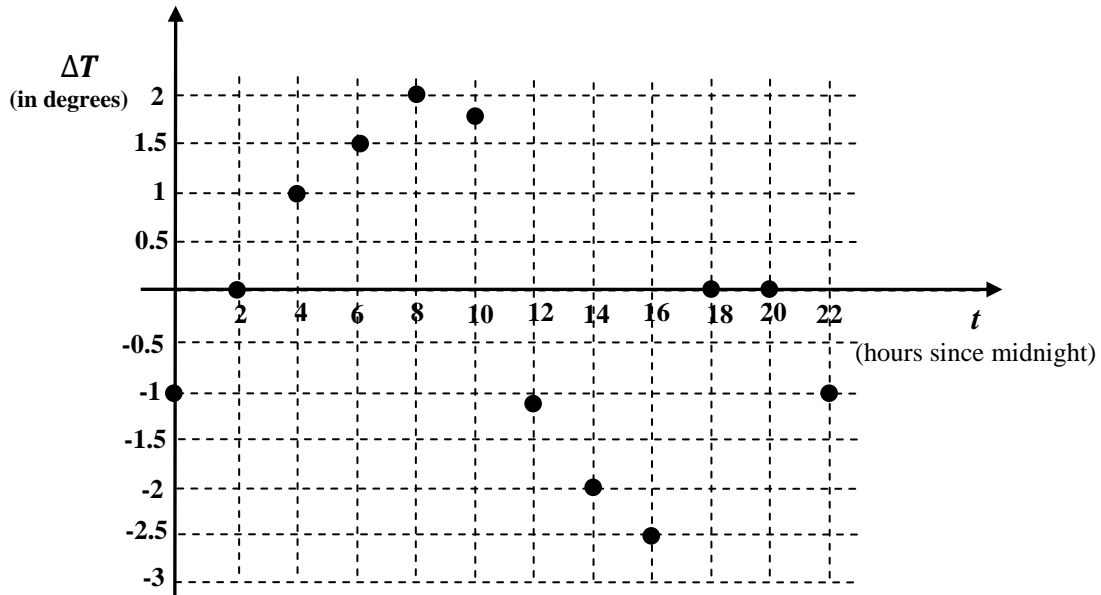
ANSWER:  $q =$ 9.2 $$  hundred bottles

c) (4pts) Your fixed costs are \$100. What is your total cost for producing 9 hundred bottles? Give your answer in dollars.

Read on graph:  $AVC(9) = 3$  dollars/bottle.  
 $VC(9) = AVC(9) * 9 = 3$  dollars/bottle \* 9 hundred bottles = 27 hundred dollars = 2700 dollars  
 $TC(9) = VC(9) + FC = 2700 + 100 = 2800$  dollars

ANSWER:  $TC(8) =$ 2800 $$  dollars

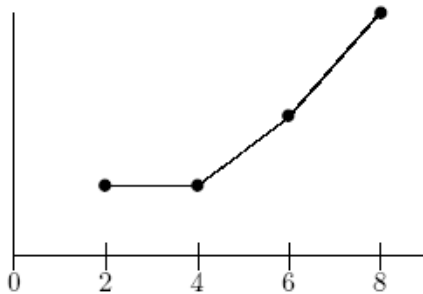
**3** (13 pts) The following graph shows the **change in temperature**, in degrees Farenheit, every two hours, over a 24-hour period beginning at midnight. Each dot on the graph represents the change in temperature over the *next* two hours.



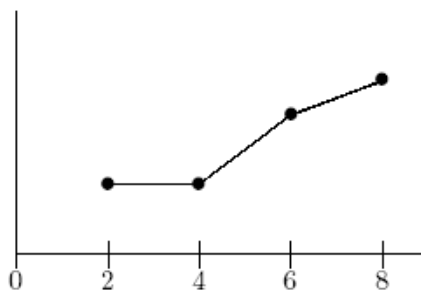
a) (2pts) What was the change in temperature from 4am to 8am?  
=1+1.5

ANSWER: 2.5 degrees

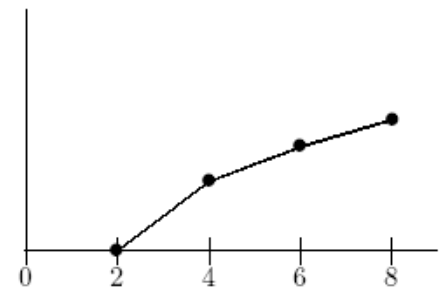
b) (4pts) Which of the following three graphs most closely resembles the **temperature** graph from t=2 to t=8? Circle one, no justification needed.



(i)



(ii)



(iii)

c) (2pts) For each of the following statements, **circle True (T) or False (F)**. No need to show work.

**T**  **F** The temperature is higher at t=10 than at t=12.

**T** **F** The temperature is higher at t=16 than at t=18.

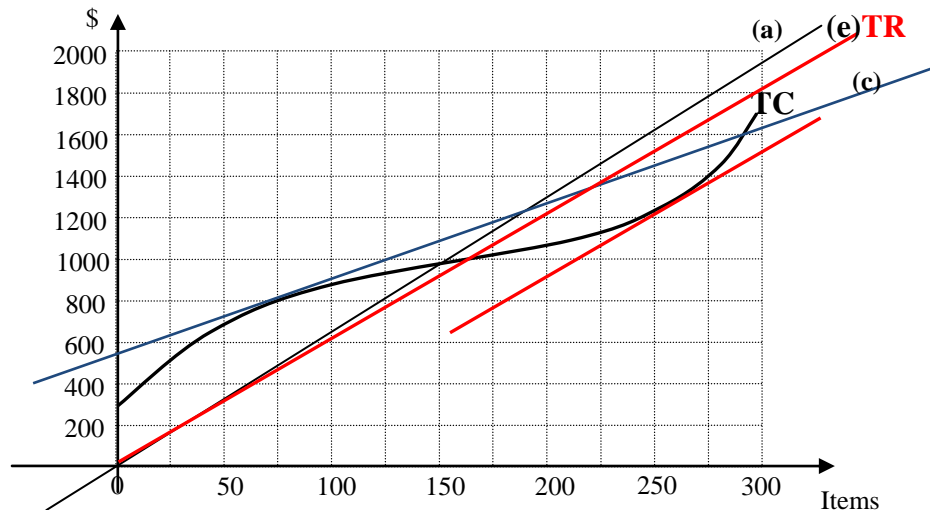
d) (5pts) Compute the average rate of change of temperature from midnight to 10 am. Show work, and correct units.

$$\Delta T = -1 + 0 + 1 + 1.5 + 2 = 3.5$$

$$\text{Average Rate} = \frac{\Delta T}{\Delta t} = \frac{3.5 \text{ degrees}}{10 \text{ hours}} = 0.35 \frac{\text{degrees}}{\text{hr}}$$

ANSWER: 0.35 Units: degrees/hour

4 (15 points) The following graph shows the total revenue (TR), in dollars, for producing Items.



a) What is the Average Cost from selling 150 Items?

$$AC(150) = \frac{TC(150)}{150} \approx \frac{970}{150} \approx 6.47 \text{ \$/item}$$

(or: slope of diagonal line thru the graph at  $q=150$  items  $\approx 1800/275 = 6.53$ )

(range: [6.3, 6.7])

Answer:  $AC(150) = \underline{6.5}$  \$ per Item.

b) What is the Average Variable Cost from selling 75 Items?

$$AVC(75) = \frac{VC(75)}{75} = \frac{TC(75) - FC}{75} = \frac{800 - 300}{75} = \frac{500}{75} \approx 6.67$$

(Or: slope of line from y-intercept to  $q=75$ )

Answer:  $AVC(75) = \underline{6.67}$  \$ per Item.

c) How much does it cost to produce the 76<sup>th</sup> item?

This is  $MC(75)$ , which is measured by the slope of tangent line at  $q=75$ .

$$\frac{1200 - 800}{175 - 75} = \frac{400}{100} = 4$$

(range: [3, 5])

Answer:  $\underline{4}$  \$ (per Item).

d) Find an interval of quantities that's 50 items long, over which the marginal cost MC is increasing but the average cost AC is decreasing.

AC: slopes of diagonals decrease up to 250 items

MC: slopes of tangents increase from 150 items onward

Pick an interval of length 50 between 150 and 250. For example:

Answer: from  $\underline{175}$  items to  $\underline{225}$  items

e) Suppose each item sells at a market price of \$6. Which quantity maximizes your profit?

Draw TR: diagonal line of slope \$6/item. Find largest difference TR-TC, with TR above TC (look for parallel tangent lines: see graph)

(range: [250, 270])

Answer: Maximum profit is obtained at  $q = \underline{260}$  Items