NAME: ________________________________ Student ID #: _________________

QUIZ SECTION:______

Math 111
Midterm I, Lecture A, version 1 -- Solutions
January 30th, 2007

Problem 1	4	
Problem 2	6	
Problem 3	20	
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Total:	**50**	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 5 pages in total and 4 problems. Check that your test is complete!
- You **must explain how you get your answers**. Correct (or incorrect) answers with no supporting work may result in little or no credit. **On problems in which you use a graph, draw any lines you use, label them, and mark points clearly.**
- Write your **final answer in the indicated spaces**.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

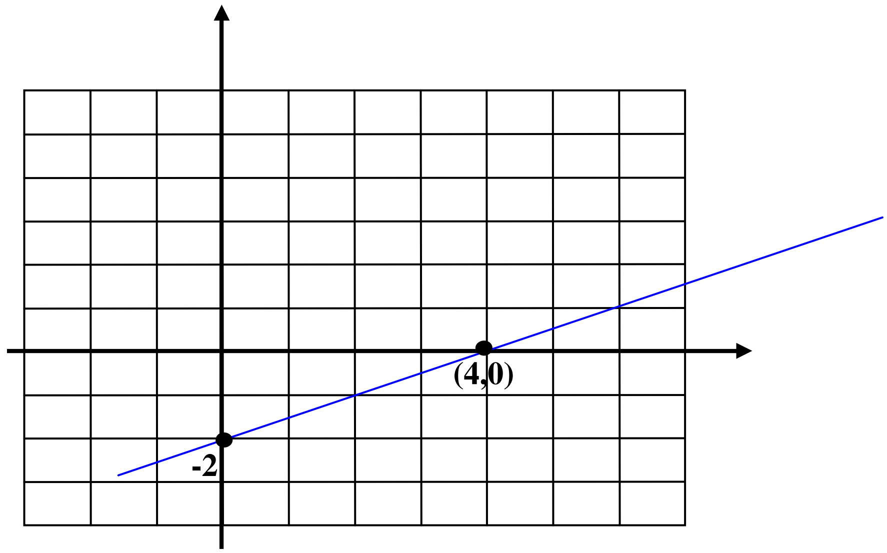
GOOD LUCK!

Do you want me to post your grade so far on the class website under the last 4 digits of your student number?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

☐ *No, please don't post my grade so far.*

1 **(4 points)** Draw the graph of the following function: $f(x) = 0.5x - 2$.
Label its y-intercept and the coordinates of one other point on the graph. No need to show any other work.



2 **(6 points)** Let *D(t)* represent the distance traveled by a car (in miles), up to time t (in hours), starting from the car's initial position (i.e. $D(0)=0$).

a) Translate the following statement into **English** (including the appropriate units):

$$D(5) - D(3) > 150 .$$

Answer:

The car traveled more than 150 miles from 3 hours to 5 hours.

b) Translate the following statement into **functional notation**:

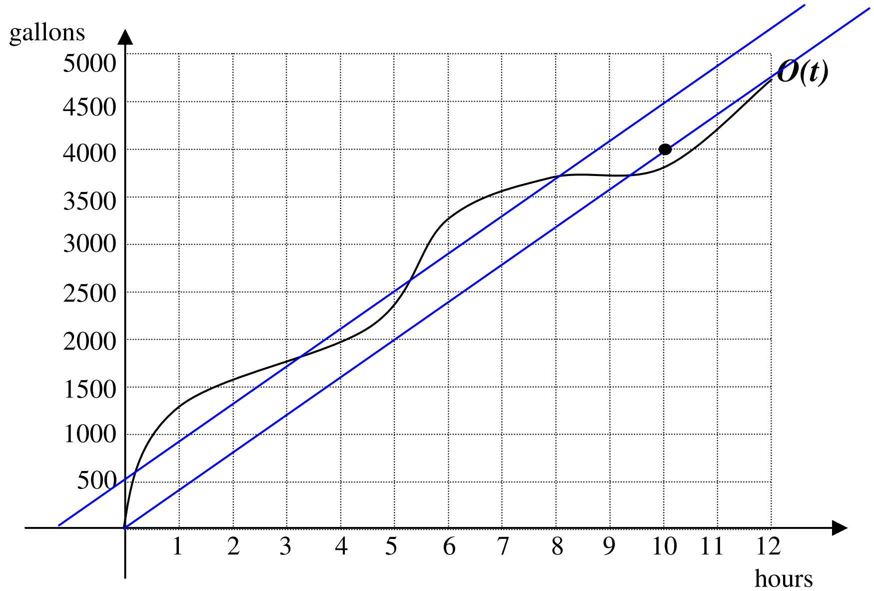
"The average trip speed of the car over the first half an hour was 60 mph"

Answer:

$$\frac{D(0.5)}{0.5} = 60$$

3 (20 points)

The graph below is of the total amount of the water ***O(t)*** which was drawn **out** of a reservoir by various times *t*, over a 12 hour interval, starting at noon.



a) How much water was drawn out of the reservoir from t = 3 pm to t = 6 pm?

Work:

O(6)-O(3)=3250-1750=1500

Answer: __1500________ gallons.

b) Find a 3-hour time interval over which the (incremental) average rate of water being drawn out of the reservoir was 400 gallons per hour.

Work:

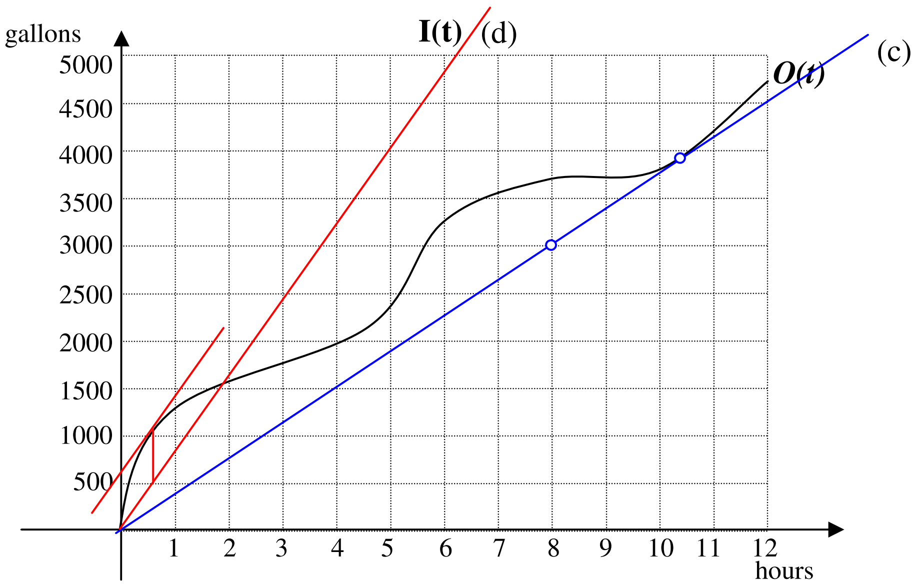
Draw a reference line of slope 400 (thru point (10, 4000)).
Move ruler parallel to it until it intersects the O(t) graph in two points which are 3 hours apart.

There are multiple correct answers:

- *from about 0.3 to 3.3*
- *from about 2.6 to 5.6*
- *from about 5.2 to 8.2*

Answer: From t =__0.3____ until t =____3.3____ hours past noon.

The following questions continue the problem from the previous page. For your convenience, here is the same graph again. Recall that ***O(t)*** is the amount of water **out** of the reservoir up to time *t*.



c) What is the lowest value of $\frac{O(t)}{t}$? At what time is it achieved?

Work:

$\frac{O(t)}{t}$ *corresponds to slopes of diagonal lines thru the graph of O(t).*

- *Draw the lowest diagonal line that touches O(t).*
- *Its slope =3000/8=375 gallons per hour,*
- *And the time this rate occurs is about t=10.4 hours.*

Answer: The lowest value of $\frac{O(t)}{t}$ is __375______ gallons per hour, at t =_10.4___ hours.

d) A pipe brings water **into** the reservoir at the constant rate of 800 gallons per hour. How much water should there be in the reservoir at noon in order not to run out at any time before midnight?

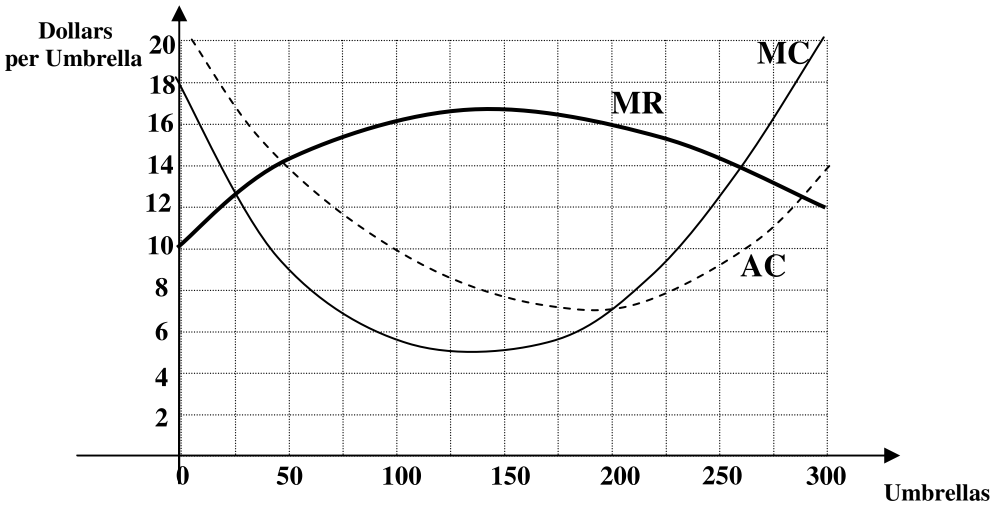
Work:

Recall that this amount equals the largest shortage.

- *Draw I(t), the total amount of water in amount: diagonal line of slope 800 (point (4000, 5)).*
- *Find the largest vertical "gap" between I(t) and O(t), with O above I.*
- *It's about 600 gallons.*

Answer: We need at least ____600_______ gallons of water in the reservoir at noon.

4 **(20 points)** The graphs below represent the **marginal cost** (**MC**), the **marginal revenue** (**MR**), and the **average cost** (**AC**) for the Seattle Rain Company, which is producing and selling Umbrellas.



a) Find the change in the total revenue if you sell 101 Umbrellas instead of 100 Umbrellas.

Work: *Recognize this as MR(100) & read it from graph*

Answer: TR(101)-TR(100)=___16____dollars.

b) What quantity of Umbrellas produced and sold maximizes the profit?

Work: *The q for where MR=MC, transitioning from MR>MC to MR<MC (in our case,that's the second crossing point of the graphs MR and MC.)*

Answer: Maximum profit is achieved at q = __260____ Umbrellas.

c) Find the breakeven price (BEP).

Work: *The y-coordinate of crossing point of AC and MC*

Answer: BEP = ___7____ Units:___$ per Umbrella___.

d) The fixed costs are FC=$150. What is the average variable cost (AVC) for producing 100 Umbrellas?

Work:

AVC(100)=VC(100)/100.

VC(100)=TC(100)-FC=TC(100)-150.

TC(100)=AC(100) x 100=10 x 100=1000

So AVC(100)=(1000-150)/100=8.5

Answer: AVC(100)= ___8.5______ dollars per Umbrella .