

NAME: _____

Student ID #: _____

QUIZ SECTION: _____

Math 111
Midterm I, Lectures B & C, Version 1 -- Solutions
 January 30th, 2007

Problem 1	4	
Problem 2	6	
Problem 3	20	
Problem 4	20	
Total:	50	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 5 pages in total and 4 problems. Check that your exam is complete!
- You **must explain how you get your answers**. Correct (or incorrect) answers with no supporting work may result in little or no credit. **On problems in which you use a graph, draw any lines you use, label them, and mark points clearly.**
- Write your **final answers in the indicated spaces**.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

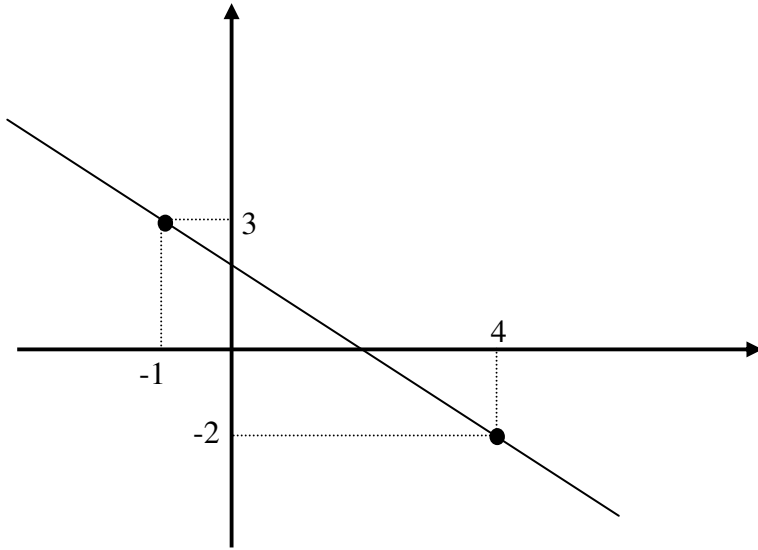
GOOD LUCK!

Do you want me to post your grade so far on the class website under the last 4 digits of your student number?

Yes, please post my grade. Sign to give permission: _____

No, please don't post my grade so far.

1 (4 points) Find the equation of the line whose graph is shown below. Put it in the form $y=mx+b$, where m and b are numbers.



$$\text{Slope: } m = \frac{-2-3}{4-(-1)} = -1$$

$$\text{Plug in a point: } 3 = -(-1)+b, \text{ so } b=2$$

Answer: $y = -x+2$.

2 (6 points)

Let $D(t)$ denote the distance (in **miles**) traveled by a car along a straight road up to time t (in **hours**), starting from the car's initial position (i.e. $D(0)=0$).

a. Translate the following statement into **English** (including the appropriate units):

$$\frac{D(5) - D(2)}{3} = 50.$$

Answer:

The average speed of the car from 2 hours to 5 hours was 50 miles per hour.

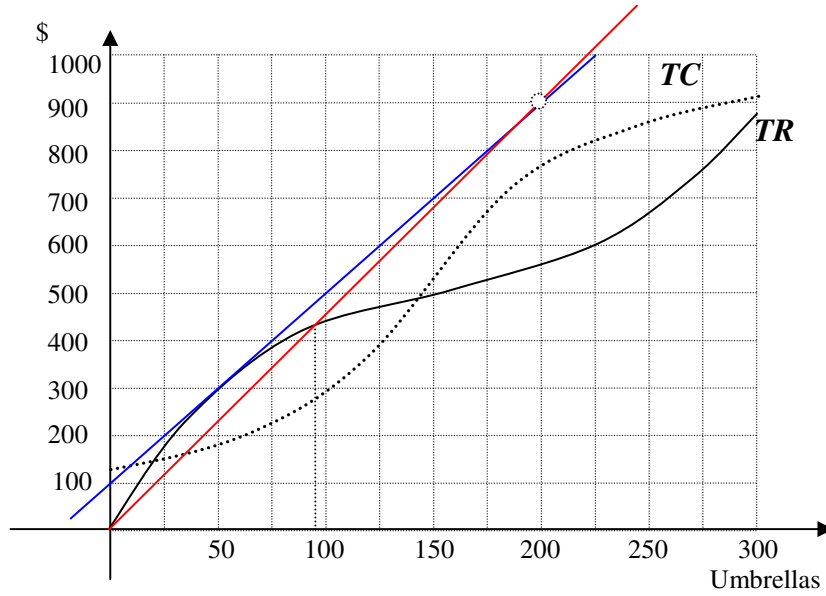
b. Translate the following statement into **functional notation**:

“The car traveled 70 miles in the first 90 **minutes**”

Answer:

$$D(1.5)=70.$$

3 (20 points) The following are the graphs of the total cost (TC) and total revenue (TR), in dollars, for the Seattle Rain Company, which is producing and selling Umbrellas.



a) What is the change in the total revenue if you sell 51 Umbrellas instead of 50 Umbrellas?

Work:

That's MR (50).

Draw the tangent line to TR at $q=50$.

*Pick 2 good points: (0,100) & (200, 900)
Slope = $800/200=4$*

Answer: 4 dollars.

b) At what quantity q of Umbrellas is the average revenue (AR) equal to \$ 4.50 per Umbrella?

Work:

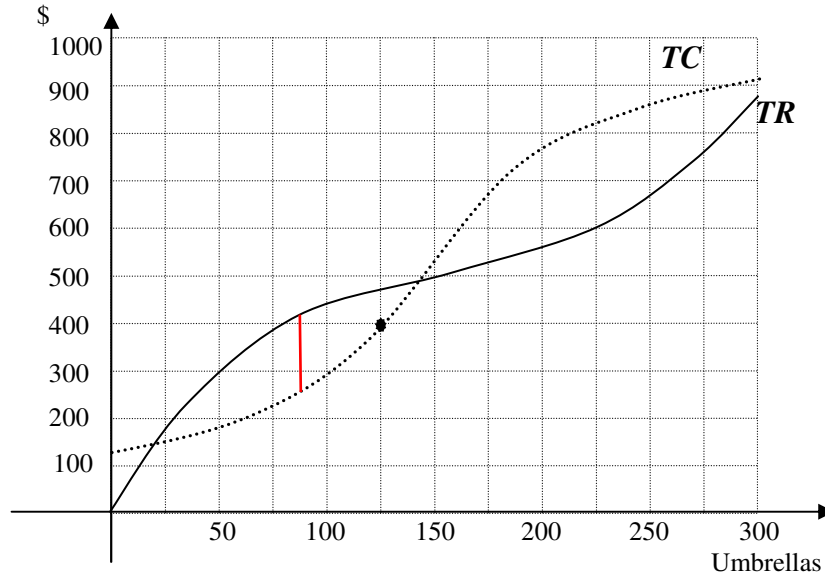
AR is measured as slopes of diagonal lines to the graph of TR

Draw a diagonal line of slope $4.5=900/200$ (from (0,0) to (200,900)).

Find the intersection points with the TR graph. It's at about 90-95 umbrellas.

Answer: At $q =$ 95 Umbrellas.

The following questions continue the problem from the previous page. For your convenience, here are the same graphs again.



c) Compute the average variable cost (AVC) of producing 125 Umbrellas.

We know that $AVC(125) = VC(125)/125$

From the graph, $TC(125) \approx \$400$, and $FC \approx \$125$, so $VC(125) \approx 400 - 125 = \275

$AVC(125) \approx 275/125 = 2.2$

Answer: $AVC(125) =$ 2.2 Units: \$ per Umbrella

d) What is the maximum profit, and at what quantity of Umbrellas produced and sold is it achieved?

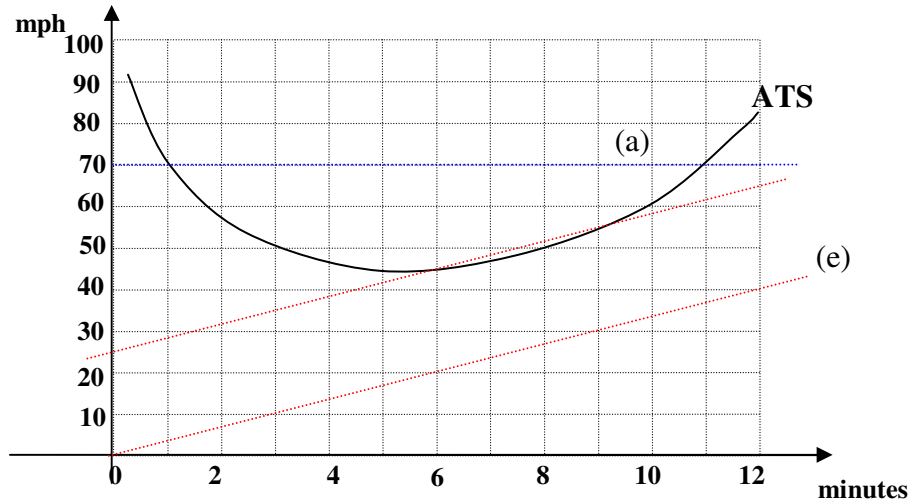
Work:

Look for the max vertical distance between TR and TC, with TR above TC.

It's about \$180, at about $q = 85$ (between 75 and 90).

Answer: the maximum profit is \$180 dollars, at $q =$ 85 Umbrellas.

4 (20 points) The graph below is of the average trip speed (ATS) for a car, in miles per hour, over a 12 minute period.



Note: In this problem, the time t is in minutes, while the ATS is in mph. To multiply, you need the units to match, so the minutes must be converted to hours in parts (b), (c) & (d) below. We took off 1 point overall for not doing this.

a) Find the first time t when the average trip speed of the car is 70 mph.

Work: $ATS=70$:

- draw horiz line at height 70,
- find first intersection point with the graph.

Answer: At $t = \underline{1}$ minutes.

b) How far has the car traveled in the first 8 minutes?

Work: $8 \text{ minutes} = 8/60 \text{ hrs} = 0.1333... \text{ hrs}$
 $ATS(8) \times 0.13333... = 50 \text{ miles/hr} \times 0.1333... \text{ hrs} \approx 6.67 \text{ miles}$

Answer: $\underline{6.67}$ miles.

c) How far did the car travel from $t = 3$ to $t = 8$ minutes?

Work:

*From the graph, ATS at both $t=3$ and $t=8$ is 50mph.
 Up to $t=3$, the car went $D(3) = 50 \times 3/60 = 2.5$ miles
 Up to $t=8$, the car went $D(8) = 50 \times 8/60 = 6.66... \text{ miles}$
 So the car traveled $6.66... - 2.5 = 4.166... \text{ miles}$*

Answer: $\underline{4.17}$ miles .

d) What was the car's average speed from $t = 3$ to $t = 8$ minutes?

Work:

$$AS = \frac{\Delta D}{\Delta t} = \frac{4.166...}{5/60} = 50$$

Answer: $AS = \underline{50}$ mph.

e) Find a three-minute time interval over which the ATS increased by 10 mph.

Work: *Draw a reference line of slope 10/3 (through point (3, 10)).
 Move ruler parallel to it until it crosses graph at two points 3 minutes apart.*

Answer: from $t = \underline{6}$ to $t = \underline{9}$ minutes.