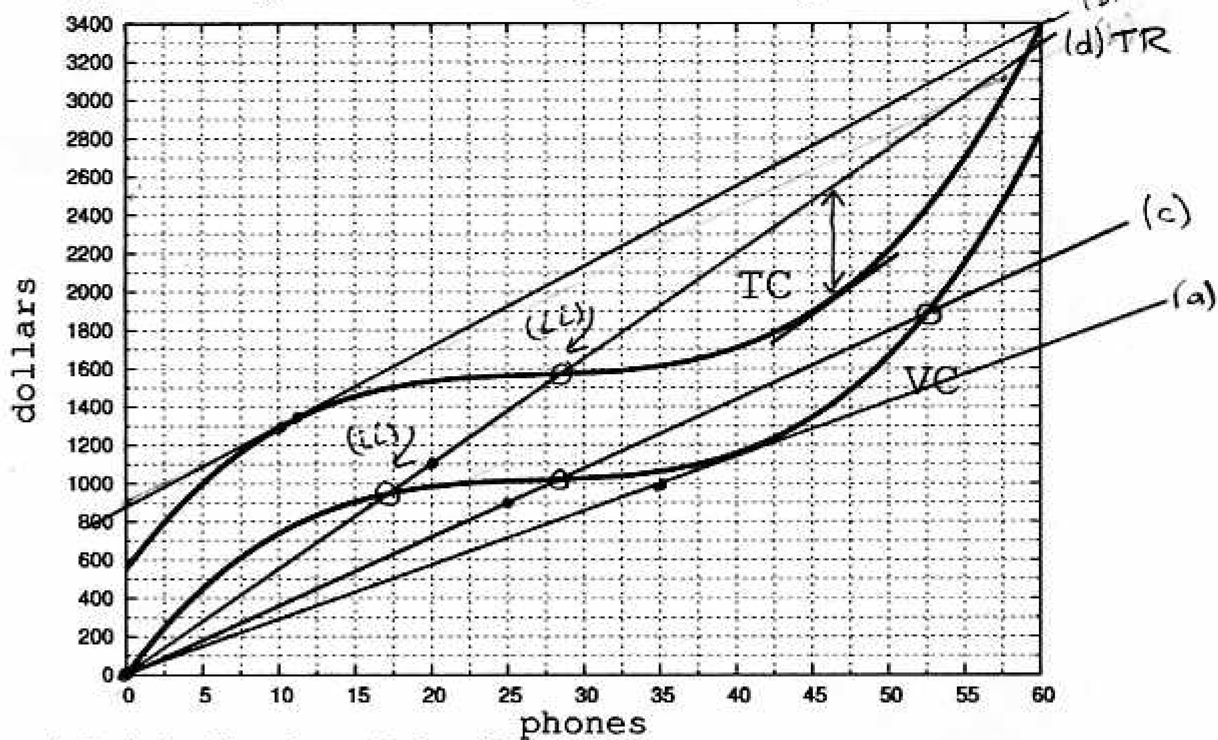
VERSION 1

1. (17 points) Shrek produces and sells mobile phones. The graphs of **total cost** and **variable cost** for producing phones are given. The x-axis is in phones and the y-axis in in dollars.



(a) (3 pts) Find the **Shutdown Price** (SDP).

SDP = slope of lowest diag. line to VC ≈ $\frac{1000-0}{35-0} = 28.57$

(0,0) (35,1000)

$SDP =$ 28.57 [26.5, 29.5] dollars per phone

(b) (3 pts) Find the **marginal cost** at $q = 10$ phones.

MC(10) = slope of the secant to TC from 10 to 11 ≈ $\frac{3400-900}{60-0} \doteq 41.\overline{6}$

(0,900) (60,3400)

$MC(10) =$ 41.67 [37, 45] dollars per phone

(c) (3 pts) Find all quantities at which **average variable cost** is \$36.00 per phone.

"slope of diag. line to VC at q" = 36 → (10, 360) ×36 (25, 900) ×36

Draw reference line with slope = 36

Find intersection with VC.

$q =$ 28 AND 52 [27,29] [51,53] phones

(d) Suppose the market price is \$55.00 per phone.

i. (2 pts) Sketch and label the graph of **total revenue** on the axes above. (20, 1100) ×55

ii. (3 pts) Find the largest interval over which Shrek's profit is negative, but his losses are less than **fixed costs**. (That is, these are the quantities when he losses money, but he should still stay open.)

want interval when TR is between VC and TC.

$q =$ 17 [17, 18] to $q =$ 28 [27, 29] phones

iii. (3 pts) Find the value of the maximum profit.

MAX PROFIT = size of the largest gap when TR is above TC

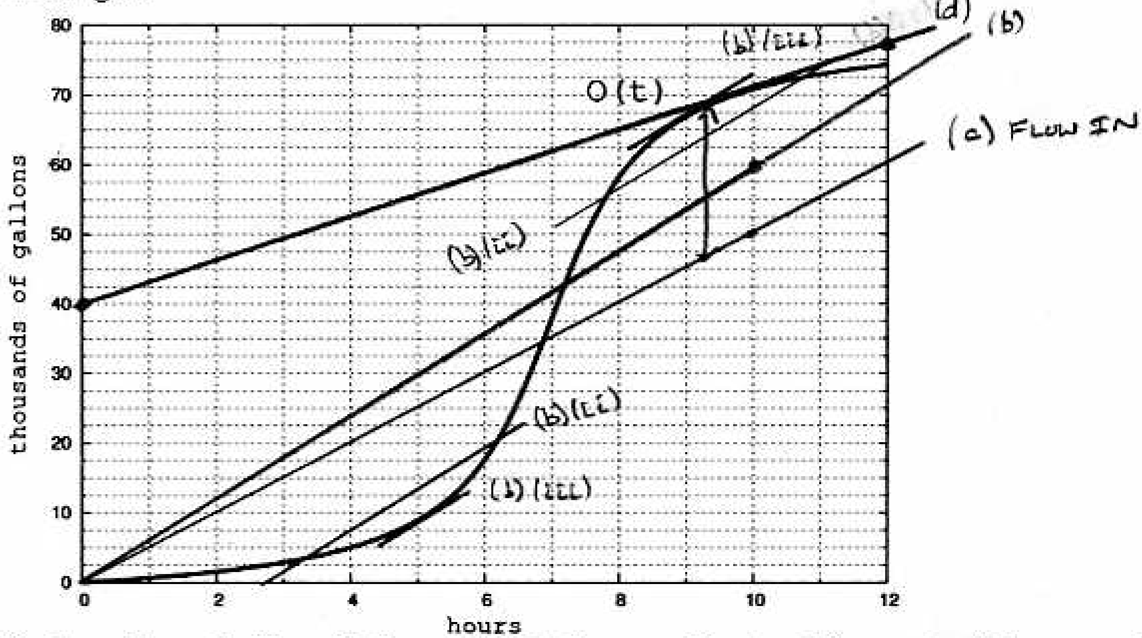
Match TR/TC slopes to find when this occurs.

Occurs at around q = 47

PROFIT = TR − TC ≈ 2550 − 1975 = 575

575 [550, 600] dollars

2. (16 points) A town is using water from a reservoir that is being refilled by an aquaduct. The graph below shows the total water drawn from the reservoir, $O(t)$, in thousands of gallons at time, t, hours after midnight.



(a) (3 pts) Find an interval of length 2 over which the overall rate of flow out of the reservoir changes from increasing to decreasing.

The slopes of diagonal lines to $O(t)$ increase from $t=2$ to $t \approx 8.8$ ← location of highest diag. slope then the slopes decrease.

$t =$ 8 to $t =$ 10 hours

or any 2-hour interval including 8.8

(b) Consider the following equation: $\dfrac{O(t+3)-O(t)}{3} = 6.$

i. (2 pts) Translate the equation into English using appropriate units.

"THE RATE OF FLOW OUT OF THE RESERVOIR OVER A 3-HOUR INTERVAL STARTING AT t HOURS IS 6 THOUSAND GALLONS PER HOUR."

ii. (3 pts) Find all times, t, for which the equation is true.

$t = 3.2$ to $t+3 = 6.2$
$t = 7.8$ to $t+3 = 10.8$

Draw reference line with slope $= 6$. → $(10, 60)$ ×6

Slide ruler parallel to 3-hour interval on $O(t)$.

$t =$ 3.2 AND 7.8 hours
[3, 3.5] [7.5, 8]

iii. (2 pts) Find all times, t, when $\dfrac{O(t+0.1)-O(t)}{0.1} = 6.$

Slide ruler parallel to 0.1-hour interval (looks like a tangent).

$t =$ 5 AND 9 hours
[4.8, 5.2] [8.8, 9.2]

(c) (3 pts) Suppose the aquaduct was filling the reservoir at a constant rate of 5 thousand gallons per hour. How much water would there have to be in the reservoir at midnight for the town to get all the water it needed for the 12 hours shown?

Refrence slope = 5 $(10, 50)$ ×5

largest gap when IN < OUT $\approx 68 - 46 =$ 22 thousand gallons
[20.5, 23]

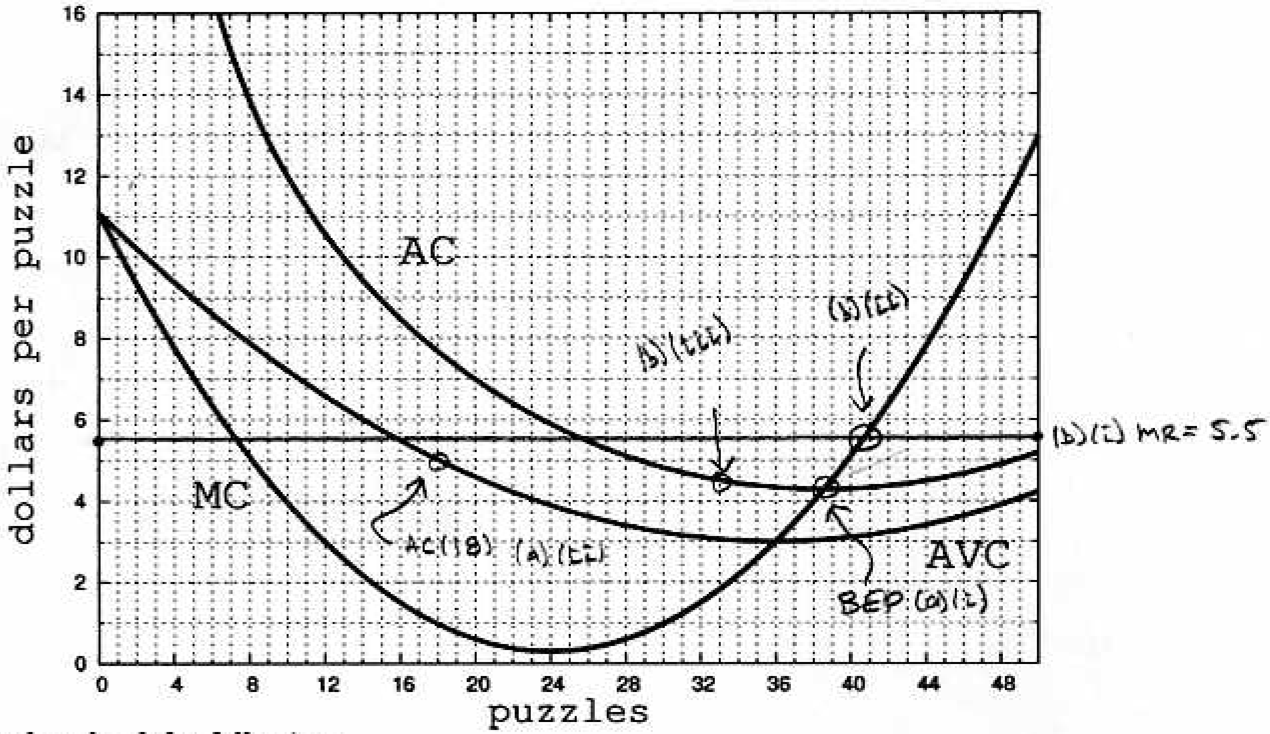
(d) (3 pts) Suppose the reservoir had 40 thousand gallons at midnight and was being filled by the aquaduct at a constant rate. How small could that rate be and provide enough water for the town for the 12 hours shown?

Find slope of line starting at 40 and staying just above the $O(t)$ graph $\approx \dfrac{77.5-40}{12-0} = 3.125$

$(0,40)$ $(12, 77.5)$

3.125 thousand gallons per hour
[2.9, 3.25]

3. (17 points) Danny is producing and selling a particular jigsaw puzzle. Below are the graphs of **marginal cost, average cost,** and **average variable cost** for producing puzzles.



(a) Find each of the following:

i. (3 pts) The Break Even Price (BEP)

BEP = "height of intersection of MC and AC" ≈ 4.20

BEP = 4.20 [4.1, 4.4] dollars per puzzle

ii. (3 pts) The **variable cost** at $q = 18$ puzzles.

VC(q) = q×AC(q) so VC(18) = 18×AC(18) ≈ 18 × 5.00 = 90

90 [88, 92] dollars

iii. (3 pts) A 3-hour interval over which $TC(q+1) - TC(q)$ decreases and then increases.

MC(q) = TC(q+1) − TC(q) MC is decreasing from 0 to 24
MC is increasing after 24

$q =$ 22.5 to $q =$ 25.5 puzzles

or ANY INTERVAL INCLUDING 24 OF LENGTH 3

(b) Suppose the **market price** is \$5.50 per puzzle

i. (2 pts) Sketch and label the graph of **marginal revenue** on the axes above. HORIZ. LINE AT 5.5

ii. (3 pts) Find the quantity at which profit is maximized.

The transition from MR>MC, to MR=MC, to MR<MC occurs at q ≈ 40.5 which we round UP to 41

$q =$ 41 puzzles

iii. (3 pts) If Danny sells exactly 33 puzzles at this market price, what is his profit?

PROFIT = TR(33) − TC(33) ⟶ ≈ 181.50 − 148.50 = 33

TR(33) = (PRICE) × (QUANTITY) = 5.50 × 33 = \$181.50
TC(33) = AC(33) × 33 ≈ 4.50 × 33 = \$148.50

33 [29.5, 36.5] dollars