

MATH 111A - Autumn 2001  
EXAM II - Version 1

1. (a) We need to find the equation of the line through the points  $(10, 62.50)$  and  $(15, 43.75)$ .

$$\text{slope} = \frac{62.50 - 43.75}{10 - 15} = \frac{18.75}{-5} = -3.75.$$

The equation of the line is  $p - 62.50 = -3.75(q - 10)$  or, solving for  $p$ ,

$$p = -3.75q + 100.$$

- (b) We are given the formula for Average Cost and  $TC = q \cdot AC$ . So,

$$TC(q) = q \left( \frac{150}{q} + 25 \right).$$

So,  $TC(3) = 3\left(\frac{150}{3} + 25\right) = \$225$ .

- (c)  $TC(q) = 150 + 25q$  and  $FC = TC(0) = 150 + 2(0) = 150$ .  $FC = \$150$ .  
(d) Profit is  $TR - TC$ . We found  $TC$  in part (b). Total Revenue is  $p \cdot q$ , where  $p$  is the price function we found in part (a).

$$TR = (-3.75q + 100)q = -3.75q^2 + 100q.$$

So, profit is given by the formula

$$\text{Profit} = -3.75q^2 + 75q - 150.$$

- (e) The graph of the profit function is a parabola. The maximum profit occurs at the vertex of this parabola. By the vertex formula, the profit occurs at  $q = \frac{-75}{2(-3.75)} = 10$ . So, the maximum profit occurs when  $q = 10$ .

2. (a)  $25 - 3t = 12 \Rightarrow t = 4.33$  hours  
(b)  $a = 25$  and  $b = 3$ . So, the amount of water in the tank at time  $t$  is

$$W(t) = 25t - 1.5t^2.$$

After 6 hours, the amount in the tank is  $W(6) = 25(6) - 1.5(36) = 96$  gallons.

- (c) We want to know when  $W(t) = 16$ :

$$25t - 1.5t^2 = 16 \Rightarrow 0 = 1.5t^2 - 25t + 16.$$

Use the quadratic formula to solve this equation:

$$t = \frac{25 \pm \sqrt{(-25)^2 - 4(1.5)(16)}}{2(1.5)} = \frac{25 \pm 23}{3}.$$

The two times are  $t = \frac{2}{3}$  and  $t = 16$ . We were asked to find a time between 0 and 8 hours. So, the answer is  $t = \frac{2}{3}$  hours.

3. (a)

$$R(q+1) = -2(q+1)^2 + 20(q+1) + 3 = -2q^2 + 16q + 21$$

$$R(q+1) - R(q) = (-2q^2 + 16q + 21) - (-2q^2 + 20q + 3) = -4q + 18$$

$$MR(q) = -4q + 18$$

(b) We need to find where  $MR = MC$ :

$$-4q + 18 = 6q + 3 \Rightarrow q = 1.5.$$

Profit is maximized at the first whole number after  $MR = MC$ , at  $q = 2$ .

(c) Profit is given by

$$TR - TC = (-2q^2 + 20q + 3) - (3q^2 + 5) = -5q^2 + 20q - 2.$$

From part (b), maximum profit occurs when  $q = 2$ . So, the maximum profit is

$$-5(2^2) + 20(2) - 2 = \$18.$$

(d)  $AC = \frac{TC}{q} = \frac{3q^2 + 5}{q} = 3q + \frac{5}{q}$ . At  $q = 10$ ,  $AC = 3(10) + \frac{5}{10} = \$30.50$ .

(e) We need to know the formula for  $AVC$ . We know that  $AVC = \frac{VC}{q}$ . We also know that  $VC = TC - FC$ .  $FC = TC(0) = 3(0)^2 + 5 = 5$ , which means that  $VC(q) = 3q^2$ . We then have  $AVC = \frac{3q^2}{q} = 3q$ . Set this equal to \$21 and solve for  $q$ .  $AVC = \$21$  when  $q = 7$ .