MATH 111D - Autumn 2001
EXAM II - Version 1

1. (a) Solve $S(t) = 50$ for t. $2t + 17.6 = 50 \Rightarrow t = 16.2$ seconds.

   (b) $m = 2$ and $k = 17.6$. So, $D(t) = t^2 + 17.6t$ and $D(20) = 752$ feet.

   (c) The interval will be from $t = 3$ to $t = 3+12 = 15$. $D(15) = 489$ and $D(3) = 61.8$ (using the formula from part (b)). Roy's average speed over this time interval is

$$\frac{D(15) - D(3)}{12} = \frac{427.2}{12} = 35.6 \text{ feetpersecond.}$$

   (d) Solve $D(t) = 1428$ for t:

$$t^2 + 17.6t = 1428 \Rightarrow t^2 + 17.6t - 1428 = 0 \Rightarrow q = 30 \text{ or } -47.6.$$

   Roy is 1428 feet from the web after 30 seconds.

2. (a) $TR(q) = pq = (100 - 3.75q)q = 100q - 3.75q^2$. So, $TR(5) = \$406.25$.

   (b) $TC(q) = AC \cdot q = \left(\frac{150}{q} + 25\right) q = 150 + 25q$. Set this equal to 300 and solve for q: $150 + 25q = 300 \Rightarrow q = 6$.

   (c) profit $= TR(q) - TC(q) = -3.75q^2 + 75q - 150$. This is a quadratic function. Its graph is a parabola that opens down. The maximum occurs at the vertex: $q = \frac{-75}{2(-3.75)} = 10$. Profit is maximized when $q = 10$.

   (d) Plug $q = 10$ into the formula for profit in part (c): profit $= -3.75(10)^2 + 75(10) - 150 = \$225$.

3. (a) $f(x+1) - f(x) = [3(x+1)^2 - 4(x+1) + 5] - [3x^2 - 4x + 5] = [3(x^2 + 2x + 1) - 4(x+1) + 5] - [3x^2 - 4x + 5] = 3x^2 + 6x + 3 - 4x - 4 + 5 - 3x^2 + 4x - 5 = 6x - 1$

   (b) $f(0) = 5 \Rightarrow f(x) - f(0) = 3x^2 - 4x \Rightarrow \frac{f(x) - f(0)}{x} = 3x - 4$. Set this equal to 15 and solve for x: $x = \frac{19}{3}$.

   (c) We are given the formula for $f(x)$ and the fact that f and g have the same value at $x = 0$ and at $x = 2$. So, $g(0) = f(0) = 5$ and $g(2) = f(2) = 9$. That means that the points $(0, 5)$ and $(2, 9)$ are on the line whose equation we want to find. The slope of that line is, therefore, $m = \frac{9-5}{2-0} = 2$ and its y-intercept is 5. Thus, $g(x) = 2x + 5$.

   (d) $g(x) - f(x) = (2x + 5) - (3x^2 - 4x + 5) = -3x^2 + 6x$. Set this equal to 1 and use the quadratic formula to solve for x: $x = 1.82$ or $x = 0.18$.

   (e) $g(x) - f(x) = -3x^2 + 6x$. This is a quadratic function that opens down. Its maximum occurs at the vertex: $x = \frac{-6}{2(-3)} = 1$. Plug this value of x back into the formula for $g(x) - f(x)$ to get its maximum value: $-3(1)^2 + 6(1) = 3$. The maximum value of $g(x) - f(x)$ is 3.