MATH 111 A, B
Exam II - Version 1 - Solutions

1. (a) (4 points) The line goes through the points $(2, 10)$ and $(16, 3)$. The slope is $m = \frac{10-3}{2-16} = -\frac{1}{2}$. So, the equation of the line is $p - 10 = -\frac{1}{2}(q - 2)$ or $p - 3 = -\frac{1}{2}(q - 16)$. Solving for p gives

$$p = -\frac{1}{2}q + 11.$$

   (b) (3 points) $TR = p \cdot q$. So, $TR = (-\frac{1}{2}q + 11)q = -\frac{1}{2}q + 11q$.

   (c) (3 points) The graph of TR is a parabola. The maximum TR occurs at the vertex of this parabola. $q = \dfrac{-11}{2(-\frac{1}{2})} = 11$.

   (d) (3 points) $FC = TC(0)$. $TC = AC \cdot q$. $TC = (\frac{10}{q} + 4)q = 10 + 4q$. $FC = TC(0) = 10$.

   (e) (4 points) Profit is $TR - TC$. At $q = 7$, TR =\$52.5 and TC =\$38. So, Profit is \$14.50.

2. (a) (3 points) The purple car is farthest ahead when the speed graphs intersect.

$$.25t + 20 = -.5t + 65 \Rightarrow .75t = 45 \Rightarrow t = \frac{45}{.75} = 60.$$

   (b) (4 points) $R(t) = .125t^2 + 20t$. $P(t) = -.25t^2 + 65t$.

   (c) (3 points) $R(5) = 103.125$

   (d) (4 points) $-.25t^2 + 65t = 7 \Rightarrow .25t^2 - 65t + 7 = 0$ Use the quadratic formula and take smallest positive solution: $t = 0.11$.

   (e) (3 points) $-.25t^2 + 65t = .125t^2 + 20t + 3$

3. (a) (3 points) $f(x)$ increases from $x = 2$ to its vertex. The vertex occurs at $x = \frac{-12.6}{2(-1)} = 6.3$.

   (b) (4 points) $\frac{f(2)}{2} = \frac{30.51}{2} = 15.755$.

   (c) (4 points) $f(0) = 10.31 \Rightarrow f(x) - f(0) = -x^2 + 12.6x \Rightarrow \frac{f(x)-f(0)}{x} = -x + 12.6$.

   (d) (5 points) $f(r+3) = -(r+3)^2 + 12.6(r+3) + 10.31 = -(r^2 + 6r + 9) + 12.6(r + 3) + 10.31 = -r^2 - 6r - 9 + 12.6r + 37.8 + 10.31 = -r^2 + 6.6r + 39.11$.