Math 111 A - Autumn 2005
Exam 2
November 10, 2005

Name: Key

Section: Version 1

Student ID Number:

TA's Name:

1	15	
2	15	
3	20	
Total	50	

- You are allowed to use a calculator, a ruler, and one **hand-written** 8.5 by 11 inch page of notes.
- Your exam should consist of this cover sheet, followed by 3 problems on 4 pages. Check that you have been given a complete exam.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Put your name on your sheet of notes and turn it in with the exam.
- You have 50 minutes to complete the exam.

GOOD LUCK!

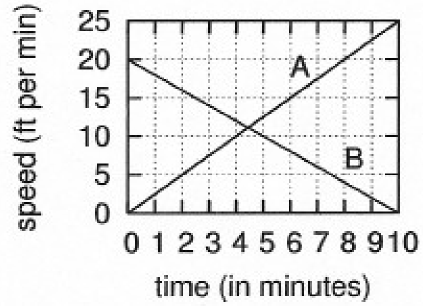
1. (15 points)

The graphs to the right give the instantaneous *speed* vs. time for two cars (Car A and Car B). The speed of Car A and Car B are given by the formulas

$$S_A(t) = 2.5t$$

$$S_B(t) = 20 - 2t.$$

At time $t = 0$, the two cars are next to one another.



(a) At what time in the first 8 minutes are the two cars furthest apart?

$2.5t = 20 - 2t$

$4.5t = 20$

$t = \frac{20}{4.5} = 4.\overline{4}$

+3

ANSWER: $t =$ 4.44 min

(b) In general, if a car has a speed formula $s = mt + k$, then its distance from the start is given by $d = \frac{m}{2}t^2 + kt$. Write out the distance vs. time formulas for the two cars. Call these functions $D_A(t)$ and $D_B(t)$.

$D_A(t) = \frac{2.5}{2}t^2$

$D_B(t) = 20t - t^2$

STEP 1: $S_A(0) = 0$ $\quad S_A(t) = 2.5t$

STEP 2: $\frac{S_A(0) + S_A(t)}{2} = \frac{2.5}{2}t =$ AVE SPEED

STEP 3: $D_A(t) = (1.25t)t$

+4

STEP 1: $S_B(0) = 20$ $\quad S_B(t) = 20 - 2t$

STEP 2: $\frac{S_B(0) + S_B(t)}{2} = \frac{20 + 20 - 2t}{2} = 20 - t$

STEP 3: $D_B(t) = (20 - t)t$

ANSWER: $D_A(t) =$ $1.25t^2$ and $D_B(t) =$ $20t - t^2$

(c) Find the average trip speed for Car A at $t = 5$ minutes.

$ATS(t) = \frac{D_A(t)}{t} = \frac{1.25t^2}{t} = 1.25t$

$t = 5 \qquad ATS = 1.25(5) = 6.25$

+4

ANSWER: $ATS =$ 6.25 ft per min

(d) At what time, after $t = 0$, will Car A catch up with Car B?

$D_A(t) = D_B(t)$

$1.25t^2 = 20t - t^2$

$2.25t^2 - 20t = 0$

$t = \frac{20 \pm \sqrt{(-20)^2 - 4(2.25)(0)}}{2(2.25)}$

$= \frac{20 \pm 20}{4.5} = 0$ or $8.\overline{88}$

+4

$t(2.25t - 20) = 0$

$t = 0$ or $2.25t - 20 = 0$

$t = \frac{20}{2.25}$

ANSWER: $t =$ 8.89 min

2. (15 points) Suppose you are running a print shop. The average cost ($AC(q)$) on an order of q reams is given by the formula

$$AC(q) = 4 + \frac{10}{q}.$$

The price per item, p, changes depending on the number of reams sold. If you sell 2 reams, the price is \$14.00 per ream, and if you sell 3 reams the price is \$13.00 per ream.

(a) What is the value of your fixed cost?

$TC(q) = q\,AC(q) = 4q + 10$

$FC = TC(0) = 10$

+3

ANSWER: 10 dollars

(b) Find the linear formula for the price per ream on an order of q reams.

$p = mq + b$

$m = \frac{13-14}{3-2} = -1$

$p = -q + b$

$14 = -2 + b$

$b = 16$

+4

ANSWER: $p = -q + 16$

(c) Find the largest possible value of total revenue, $TR(q)$.

$TR(q) = pq = -q^2 + 16q$

largest possible value = height of vertex

vertex: $q = -\frac{b}{2a} = -\frac{16}{2(-1)} = 8$

$TR(8) = -8^2 + 16(8) = 64$

+4

ANSWER: 64 dollars

(d) Find the quantity that gives the maximum profit.

Profit $= TR(q) - TC(q)$

$= (-q^2 + 16q) - (4q + 10)$

$= -q^2 + 12q - 10$

vertex: $q = \frac{-12}{2(-1)} = 6$

+4

ANSWER: $q =$ 6 reams

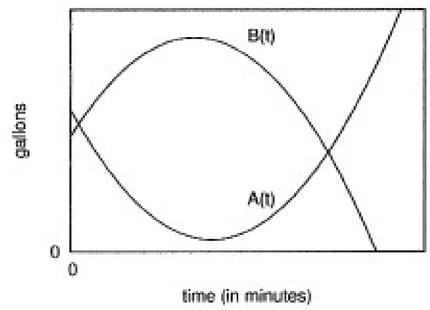
3. (20 points)

The amount of water in two vats (Vat A and Vat B) at time t is given by the formulas

$$A(t) = t^2 - 16t + 70$$

$$B(t) = -t^2 + 14t + 57$$

where t is in minutes and the amount of water is in gallons.



(a) At what time, after $t = 0$, does Vat B contain zero gallons.

$B(t) = 0$

$-t^2 + 14t + 57 = 0$ +2

$t = \dfrac{-14 \pm \sqrt{14^2 - 4(-1)(57)}}{2(-1)}$ +1 $= \dfrac{-14 \pm \sqrt{424}}{-2}$

+4

$= \cancel{-3.295630141}$ or 17.29563014

+1

ANSWER: $t =$ 17.30 min

(b) Write out the formula for $\dfrac{A(t+5) - A(t)}{5}$ and simplify as much as possible.

$A(t+5) = (t+5)^2 - 16(t+5) + 70$ +1

$= t^2 + 10t + 25 - 16t - 80 + 70$

$= t^2 - 6t + 15$ +1

$\dfrac{A(t+5) - A(t)}{5} = \dfrac{(t^2 - 6t + 15) - (t^2 - 16t + 70)}{5} = \dfrac{\cancel{t^2} - 6t + 15 - \cancel{t^2} + 16t - 70}{5}$ +1

$= \dfrac{10t - 55}{5} = 2t - 11$

+1

+4

ANSWER: $\dfrac{A(t+5) - A(t)}{5} =$ $2t - 11$

(This question is continued on the next page)

Once again, the formulas for Vat A and Vat B are

$$A(t) = t^2 - 16t + 70 \qquad B(t) = -t^2 + 14t + 57$$

(c) At what time does the water in Vat A reach its lowest level? How many gallons are in Vat A at that time?

vertex +2 $t = \frac{-(-16)}{2(1)} = 8$

$A(8) = 8^2 - 16(8) + 70 = 6$

+2

+4

ANSWER: $t =$ 8 min and $A(t) =$ 6 gallons

(d) Give the *first* time when Vat B has 25 gallons more than Vat A.

$B(t) = A(t) + 25$ +2

A (A+25) -2

$-t^2 + 14t + 57 = t^2 - 16t + 70 + 25$

$-2t^2 + 30t - 38 = 0$ +1

+4

$t = \frac{-30 \pm \sqrt{30^2 - 4(-2)(-38)}}{2(-2)} = \frac{-30 \pm \sqrt{596}}{-4}$

+1

$t = 1.396722192$ or 13.60327781

ANSWER: $t =$ 1.40 min

(e) When is the amount in Vat B above the amount in Vat A by the greatest difference?

vertex of $B(t) - A(t)$

$(-t^2 + 14t + 57) - (t^2 - 16t + 70)$

$-2t^2 + 30t - 13$ +2

+4

$t = -\frac{30}{2(-2)} = 7.5$ +2

ANSWER: $t =$ 7.5 min