NAME: _____ Student ID #: _____

QUIZ SECTION:_____

Math 111 A
Midterm II

November 9th, 2006

Problem 1	18	
Problem 2	22	
Problem 3	10	
Total:	**50**	

- You may use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 4 pages in total and 3 problems. Please check your test for completeness.
- You **must use the methods of this class to solve the problems, and you must show entirely how you get your answers**. Work done "in your head" cannot get credit. Work done by guessing and checking, or by reading off values on a graphing calculator may get little credit. Correct answers with incomplete, wrong or missing work will get partial credit at best.
- Write your **final answer in the indicated units and in the indicated spaces**.
- If you need more room, use the backs of pages and indicate to the reader that you have done so. If you still need more paper, ask your TA for some more, write your name and section on it and make sure you turn it in to your TA inside your test.
- Read each question carefully. Raise your hand if you have a question.

GOOD LUCK!

Do you want me to post your grade so far on the class website under the last 4 digits of your Student Number?

☐ *Yes, please post my grade. Sign to give permission:* _____

☐ *No, please don't post my grade so far.*

1 (18 points) A Red car starts off at North Bend and drives along the I90 highway for half an hour. Its distance from North Bend during this trip is given by the following function:

$$R(t) = -80t^2 + 80t$$

(where the time t is measured in **hours**, and the distance R is measured in miles)

a) When was the average trip speed of the Red car 60 mph?

Work:

ATS(t)=60 ==> t=?
ATS(t)=R(t)/t= - 80t+80
Solve - 80t+80=60 for t.
t=20/80=0.25

Answer: at ____0.25_____ hours

b) What was the average speed of the Red car from 15 **minutes** to 21 **minutes**?

Work:

First, convert to hrs: 15min=15/60 hrs= 0.25 hrs & 21 min= 21/60 hrs=0.35 hrs.

$$AS = \frac{R(0.35) - R(0.25)}{0.1} = \frac{18.2 - 15}{0.1} = 32$$

Answer: AS = ____32____ mph.

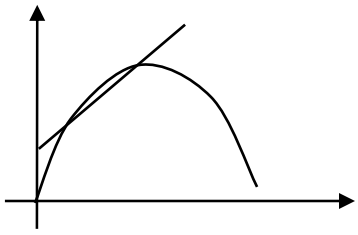
c) A Blue truck is 2 miles ahead of the Red car at time t=0, and drives along the highway in the same direction at a constant speed of 50 mph. What is the formula for $B(t)$, the distance of the Blue truck from North Bend, in terms of time t? (No work or explanation needed.)

Note: Since the speed is constant at 50 mph, the distance will be a linear function of slope = 50.
The Red car has R(0)=0, so if the Blue truck is 2 miles ahead at t=0, B(0)=y-intercept=2

Answer: $B(t)$=____**50t+2**_____

d) Does the Red car pass the Blue truck, and if so, when?

Work: The red car passes the blue truck when R(t)=B(t)
(transitioning from R(t)<B(t) to R(t)>B(t))
$-80t^2+80t=50t+2$
$-80t^2+30t-2=0$
Solve via the quadratic formula: t=0.0867 or t=0.2882.
Sketching the graphs, we see that the first solution is the correct one (the second solution corresponds to the blue truck passing the red car).



Answer: No / Yes, at t = ____0.0867_____ hours

2 (22 points) You are producing and selling bottles of Zap Energy Drink.
If you sell q **hundred** bottles, your price per bottle is given by the function:

$$p(q) = 4 - 0.2q \text{ (in dollars per bottle)}$$

Your average cost for producing q **hundred** bottles of is

$$AC(q) = 0.07q + 1.3 + \frac{4}{q} \text{ (in dollars per bottle)}$$

Caution: In this problem, please pay attention to units!

a) Write the formulas, in terms of q, for the Total Revenue, Total Cost and Profit functions. No explanation is needed, but simplify your formulas and include units.

Answer: TR(q) = $-0.2q^2+4q$ Units: hundreds of dollars

TC(q)= $0.07q^2 +1.3q+4$ Units: hundreds of dollars

Profit(q)= $-0.27q^2 + 2.7q - 4$ Units: hundreds of dollars

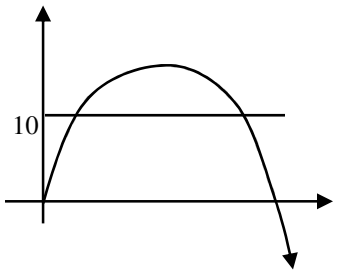
Note: The TR formula comes from p(q) x q, the TC one from AC(q) x q, and the profit one from TR-TC
The units are hundreds of dollars, since you're multiplying dollars per bottle times hundreds of bottles

b) What are all the possible quantities of bottles you can sell if you want your Total Revenue to be at least $1000?

Work:
We want TR(q) ≥10 hundred dollars. TR is a quadratic function, whose graph is a concave-down parabola. Sketch the graph, and note that TR is above 10 between the roots of the equation:

$TR=10$
$-0.2q^2+4q=10$
$-0.2q^2+4q-10=0$

Apply the quadratic formula: q=2.9289 or q=17.071



Answer: from 293 to 1707 bottles.

c) What is the largest Profit you can make, and how many bottles do you have to sell in order to achieve it?

Work:

Profit(q)= $-0.27q^2 + 2.7q - 4$ is a quadratic function, whose graph is a concave-down parabola. The largest value will occur at its vertex. Apply the vertex formula to find the q for the vertex:

$$q = -\frac{b}{2a} = -\frac{2.7}{2(-0.27)} = 5$$

The largest value is *Profit*(5)= $-0.27(5)^2 + 2.7(5) - 4=2.75$.

Answer: The largest profit is 275 dollars, and it occurs at 500 bottles.

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d) What is the longest interval of quantities over which the Profit is decreasing but the TR is increasing?

Work:

Both Profit and TR are concave-down, so they increase before the vertex, and decrease after the vertex.

So: Profit decreases after its vertex at q=5, and TR increases before its vetex at $q = -\frac{4}{2(-0.2)} = 10$.

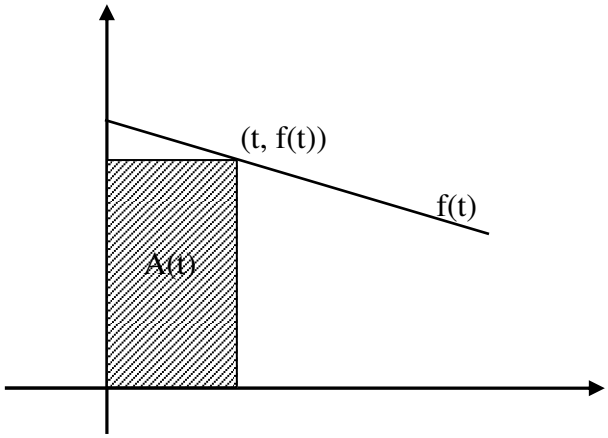
So: both Profit decreases and TR increases for q between 5 and 10.

Answer: from q=___500_____ bottles to q=___1000_____ bottles.

3 (10 points) The function $f(t)$ graphed on the right represents the average trip speed (ATS) of a car (in miles per hour, after t hours from the start of the trip). Its formula is:

$$f(t) = 60 - 16.5t$$

Define a new function, $A(t)$, to be the area of the rectangle under the graph of $f(t)$, with one side along the horizontal axis, one along the vertical axis, and the top right corner at the point of coordinates $(t, f(t))$, as shown.



a) What is the formula for $A(t)$? (in terms of t) What are its units?

Work:

$A(t)=tf(t)=t(60-16.5t)=60t-16.5t^2$

Units: we're multiplying (hrs)(miles per hour), so we get miles

Answer: $A(t)$= _____$60t-16.5t^2$_____ Units: _miles_____

b) What does this function represent?

For example, if you compute the function A at a specific time (say, $A(1)=43.5$), what does this tell you about the car?

Explain: Since we're multiplying the average trip speed by time t "so far", we get the distance covered by the car so far (t*ATS(t)=D(t))

Note: Simply re-stating your answer does not constitute an explanation. You need to say how do you know that A(t) is the distance traveled by the car. Also, the units being miles are not enough of an argument. After all, lots of things other than the distance covered by this particular car can be measured in miles!.

Answer: The function $A(t)$ represents _____the distance traveled by the car_____ after t hrs.