NAME: ________________________________ Student ID #: ________________

QUIZ SECTION:______

Math 111 D
Midterm II

November 9th, 2006

Problem 1	20	
Problem 2	20	
Problem 3	10	
Total:	**50**	

- You may use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 4 pages in total and 3 problems. Please check your test for completeness.
- You **must use the methods of this class to solve the problems, and you must show entirely how you get your answers**. Work done "in your head" cannot get credit. Work done by guessing and checking, or by reading off values on a graphing calculator may get little credit. Correct answers with incomplete, wrong or missing work will get partial credit at best.
- Write your **final answer in the indicated units and in the indicated spaces**.
- If you need more room, use the backs of pages and indicate to the reader that you have done so. If you still need more paper, ask your TA for some more, write your name and section on it and make sure you turn it in to your TA inside your test.
- Read each question carefully. Raise your hand if you have a question.

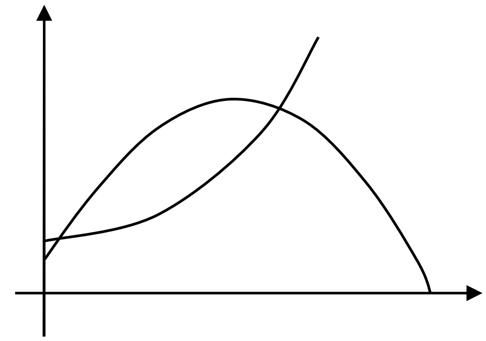
GOOD LUCK!

Do you want me to post your grade so far on the class website under the last 4 digits of your Student Number?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

☐ *No, please don't post my grade so far.*

1 (20 points) You have two bank accounts: a checking account and a savings account. The balances of these accounts (in thousands of dollars) after t days are given by:



Checking: $C(t) = -t^2 + 24t + 10$

Savings: $S(t) = 0.3t^2 + 0.2t + 13$

a) When will the balance of your checking account be $0?

C(t)=0
$-t^2+24t+10=0$
Apply the quadratic formula: t≈ - 0.41 or 24.41 (rounded to 2 decimal digits).

Answer: After t = __24.41___ days

b) Let A(t) be the total amount of money you have in both accounts combined. When will A(t) be largest?

Both accounts combined: $A(t)=C(t)+S(t)=(- t^2+24t+10)+(0.3t^2+0.2t+13)= - 0.7\, t^2+24.2\, t + 23$
This is a quadratic whose graph is a concave down parabola. It is largest at its vertex: t= - 24.2/[2(-0.7)] ≈17.29

Answer: After t = __17.29____ days

c) What is the average rate of change in the balance of your savings account from t= 2 days to t=10 days?

Average rate of change of S(t) from 2 days to 10 days: $\dfrac{S(10)-S(2)}{10-2} = \dfrac{45-14.6}{8} = 3.8$

Answer: ____3.8_______ Units:_thousand dolars per day_

OR: ______3800_______ Units:_____dollars per day ___

d) You open another savings account whose balance after t days is denoted N(t). The balance of this new account is always the same as the amount that was in the original savings account ten days before. Find a formula for N(t). Simplify until your formula is in the form $N(t) = at^2 + bt + c$.

New account now=savings account 10 days before now translates into:
N(t)=S(t-10)=
$=0.3(t-10)^2+0.2(t-10)+13=$
$=0.3(t^2-20t+100)+ 0.2(t-10)+13=$
$=0.3t^2-6t+30+0.2t-2+13=$
$=0.3t^2-5.8t+41$

Answer: N(t) = ____$0.3t^2-5.8t+41$______

2 (20 points) You produce and sell bottles of Zap Energy Drink. If you sell q **hundred** bottles, your price per bottle is given by the function:

$$p(q) = 4 - 0.2q \text{ (in dollars per bottle)}$$

Your average cost for producing q **hundred** bottles of is

$$AC(q) = 0.07q + 1.3 + \frac{4}{q} \text{ (in dollars per bottle)}$$

Caution: In this problem, please pay attention to units!

a) Write the formulas, in terms of q, for the Total Revenue, Total Cost and Profit functions. No explanation is needed, but simplify your formulas and include units.

Answer: TR(q) = _______ $-0.2q^2 + 4q$ ___________ Units: _hundreds of dollars_

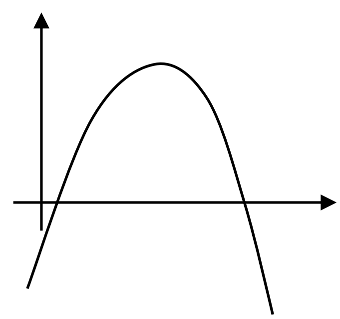
TC(q)= ______ $0.07q^2 + 1.3q + 4$ ________ Units: _hundreds of dollars_

Profit(q)=_____ $-0.27q^2 + 2.7q - 4$ ______ Units: _hundreds of dollars_

Note: The TR formula comes from p(q) x q, the TC one from AC(q) x q, and the profit one from TR-TC
The units are hundreds of dollars, since you're multiplying dollars per bottle times hundreds of bottles

b) What are all the possible quantities of bottles you can sell in order to make a (positive) profit?

Profit(q)= $-0.27q^2 + 2.7q - 4$ is a quadratic function, whose graph is a concave-down parabola.
It is positive between its roots.
We find its roots (x-intercepts) via the quadratic formula:
$-0.27q^2 + 2.7q - 4$ =0 at q=1.808 & q= 8.1914



Answer: _between 1.81 and 8.19 hundred ___bottles.
OR, better yet: Answer: _between 181 and 819 ___bottles.

c) What is the maximal value of your Total Revenue, and how many bottles do you have to sell in order to achieve it?

TR(q) = $-0.2q^2 + 4q$ is a quadratic function whose graph is a concave-down parabola.
Its max occurs at its vertex: q=-b/(2a)=-4/(2*-0.2)=10.
The max value is TR(10)=-0.2*10^2+4(10)=20.
Convert to the appropriate units.

Answer: The maximal value of TR is ___2000_____.dollars, and it occurs at _____1000___ bottles.

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d) Find the marginal revenue at q=1 **hundred** bottles.

MR(1) = TR(1.01)-TR(1)

$= [-0.2(1.01)^2 + 4(1.01)] - [-0.2(1)^2 + 4(1)]$

=3.83598-3.8

=0.03598

Answer: MR(1)=_$0.03598_.

3 (10 points) You produce and sell Gizmos. Your marginal revenue and marginal cost at q Gizmos are given by the formulas:

$$MC(q) = 2 + 0.6q\,,\ MR(q) = 5 - 0.3q$$

Your total cost, in dollars, is given by the formula $TC(q) = 0.3q^2 + 2q + 30$.

a) For what number of Gizmos will your profit be maximal?

The profit is maximized when MR=MC, transitioning from MR>MC to MR<MC

Set MR=MC and solve for q:

2+0.6q=5-0.3q

0.9q=3

q=3/0.9=3.33333…

Recall that when using this method we need to round UP to the next whole number of items.

(explained in class, included in the review file, summarized at the end of WS 14 in your text)

Answer: *For q=* ____4_____Gizmos

(*note: your answer should be a whole number*)

b) Compute the Breakeven Price.

BEP is the lowest value of AC, or the y-coordinate of the intersection point of AC & MC.

Here, AC(q)=TC(q)/q=0.3q+2+30/q. This is not a quadratic, so its lowest value is hard to determine (can't use the vertex formula!).

Use AC=MC instead.

0.3q+2+30/q=2+0.6q

Subtract 2 from both sides and then multiply both sides by q: $0.3q^2+30=0.6q^2$

Subtract $0.3q^2$ from both sides: $0.3q^2=30$.

1) You can put in standard form: $0.3q^2-30=0$ and use the quadratic formula.

2) Or, simpler, divide by 0.3:

$q^2=100$, then take square root of both sides:

Either way, q=±10. Only q=10 makes sense for a quantity.

Plug back into MC or AC to get BEP:

BEP=MC(10)=2+0.6(10)=8

Answer: BEP=$ _8________.