1. (11 pts=2+4+5) Two cars, an Audi and a Bug, start at the same place and drive in the same direction on a straight road. The **distance** covered by the **Audi** in the first *t* minutes is given by:

$$D_A(t) = 0.75t \text{ (in miles)}$$

The **average trip speed (ATS) of the Bug** during the first *t* minutes is given by the function:

$$s_B(t) = 0.08t + 0.2 \text{ (in miles per minute)}$$

a) Find the formula, in terms of t, for the distance covered by the Bug in the first t minutes.

Use: D(t) = t • ATS(t):

$$D_B(t) = ts_B(t) = t(0.08t + 0.2) = 0.08t^2 + 0.2t$$

Answer: $D_B(t)$= $0.08t^2 + 0.2t$.

b) At *t* = 2 minutes, which car is ahead, and by how many miles?

In the first 2 minutes, the Audi traveled: $D_A(2)=0.75(2)=1.5$ miles.
In the same 2 minutes, the Bug traveled: $D_B(2)=0.08(2)^2+0.2(2)=0.72$ miles.
Hence the Audi is ahead of the Bug, by 1.5-0.72=0.78 miles.

Answer: Car _**Audi**__ is ahead of Car_**Bug**_ by _**0.78**_ miles .

c) When is the Bug exactly 1 mile ahead of the Audi?

The Bug is ahead by 1 mile when the distance it traveled is larger by 1 mile.
That is, we want to find the time t when:

$$D_B(t)=D_A(t)+1$$

$$0.08t^2 + 0.2t = 0.75t + 1$$

Solve for t:

$$0.08t^2 - 0.55t - 1 = 0$$

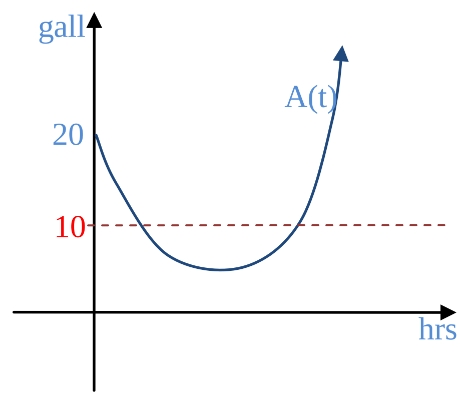
Applying the quadratic formula we get: t ≈ -1.49 or t ≈ 8.37 minutes.
Trip starts at t=0, so only the 2nd time is valid.

Answer: The Bug is 1 mile ahead of the Audi at t =___**8.37**______minutes.

2. (15 pts=6+3+6) Water is flowing in and out of a vat. The formula for the amount of water in the vat, at t hours, is: $A(t) = 0.5t^2 - 6t + 20$ gallons.

a) Sketch the graph of *A(t)* and find all the times when the vat contains **at most** 10 gallons of water.

The graph of A(t) is a concave-up parabola.
Its y-intercept is A(0)=20



A(t)=10:

$$0.5t^2 - 6t + 20 = 10$$

$$0.5t^2 - 6t + 10 = 0$$

Quadratic Formula gives: t = 2 and t = 10

Looking at the graph, we see that the graph of A(t) is at most height 10 between the two times where it's equal to 10, so from t=2 to t=10.

Answer: Vat A contains ≤10 gallons at times: _______________ **from t=2 to t=10 hours**___________.
(your answer should be one or more time intervals)

b) Another vat, vat B, contains 10 gallons of water initially. Water is added to vat B at a constant rate of 5 gallons an hour. Find a formula (in terms of *t*) for the amount of water in vat B at time *t* hours.
(You don't need to show work.)

Answer: B(t)=__**5t+10**____ gallons.

c) At what time is the **difference** between the amount of water in vat B and the amount of water in vat A the greatest?

The difference is given by the function: B(t)-A(t)=(5t+10)-(0.5t^2-6t+20)= - 0.5t^2+11t-10.

This is a quadratic function, whose graph is a concave-down parabola, so it's max at its vertex:

$$t = -\frac{11}{2(-0.5)} = 11$$

Answer: At t=___11____ hours

3. (15 pts=3+6+6) You produce Blivets in batches of 1 to 60. Each Blivet costs you $5 to produce, and your fixed costs are $100.

Your total revenue from selling *q* Blivets is: $TR(q) = -0.3q^2 + 25q$ dollars.

a) Find a formula for your Average Revenue from selling *q* Blivets. Simplify and specify the units for AR.

$$AR(q) = \frac{TR(q)}{q} = \frac{-0.3q^2 + 25q}{q} = -0.3q + 25$$

Answer: AR(q)=__**-0.3q+25**__Units:___**$ per Blivet**__

b) Recall tha $MR(q) = TR(q+1) - TR(q)$t. Find a formula, in *q*, for the Marginal Revenue *MR(q)*. Simplify as much as possible.

$$MR(q) = TR(q+1) - TR(q)$$
$$= [-0.3(q+1)^2 + 25(q+1)] - [-0.3q^2 + 25q]$$
$$= [-0.3(q^2 + 2q + 1) + 25(q+1)] - [-0.3q^2 + 25q]$$
$$= -0.3q^2 - 0.6q - 0.3 + 25q + 25 + 0.3q^2 - 25q$$
$$= \cancel{-0.3q^2} - 0.6q - 0.3 + \cancel{25q} + 25 + \cancel{0.3q^2 - 25q}]$$
$$= -0.6q + 24.7$$

Answer: *MR(q)*=____**- 0.6q+24.7**______

c) What number of Blivets maximizes your profit?

You could solve this either one of two ways:

<u>Method I:</u> Set MR=MC, and solve for q.

You have found a formula for MR in part b). Since the problem specifies that each Blivet costs you $5 to produce, MC(q)=5 for all q.

So: - 0.6q+24.7=5, which gives q=19.7/0.6≈32.833

With this method, we need to round UP to the next whole number of Blivets: q=33.

<u>Method II:</u>

Since each Blivet costs you $5 to produce, and your fixed costs are $100, your Total Cost formula is

TC(q)=5q+100.

Then the Profit is:

P(q)=TR(q)-TC(q)=(-0.3q^2+25q)-(5q+100)=-0.3q^2+20q-100.

This is a concave-down parabola, so it's max at its vertex:

q=-20/(2(-0.3))=20/0.6≈33.33

With this method, we need to round off to the nearest whole number, so to q=33.

Either method, the answer should be:

Answer: Profit will be maximal at q=__**33**___ Blivets *(answer should be a whole number)*

4. (9 pts=4+5) You are producing Items.
Your **average cost** for producing ***q* hundred Items** is given by the function:

$$\mathrm{AC}(q) = \frac{50}{q} + 3 \quad \text{(dollars per Item)}$$

a) For what number of Items is your average cost $7 per item?

AC=7

50/q+3=7

50/q=4

50=4q

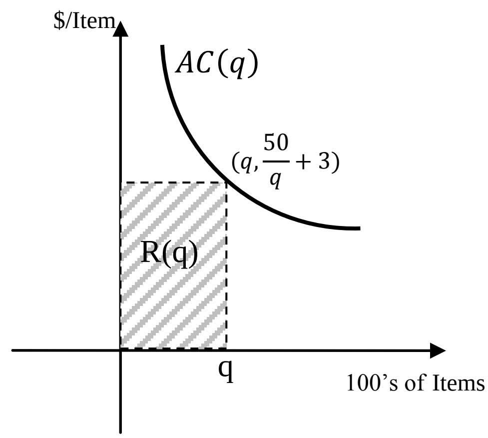
q=50/4=12.5 hundred items

Answer: For q=____**1250**________ Items.

b) Define a new function, *R(q)*, to be:

the area of the rectangle under the graph of the average cost *AC(q)*,

with a corner at the point (q, $\frac{50}{q} + 3$), and sides along the axes.



Compute *R(50)* and state what this area represents, in terms of your costs.

Include correct units.

R(50)=(base)(height)=50(50/50+3)=50+3(50)=200

Units: (100's of ~~Items~~)($ / ~~Item~~) =100's of $

R(q)= (base)(height)= (q)•AC(q)=TC(q)

Answer: *R(50)* = ___**200**___ Units:___**hundreds of dollars**____

This area represents: ___**the Total Cost**___for producing ____**5000 Items**______.