AUT 08, Nichifor, Midterm 2, v1 SOLUTIONS

1. Your company, "*RainCheck*", produces extra-large umbrellas. The selling price is $19.99 per umbrella.

Each umbrella costs you $9.50 to produce. Your fixed costs are $250.

This problem was based on Worksheet 10. Similar ideas were investigated in the Tasty Tour Activity.

a) (4 points) Write down formulas in terms of quantity *q* of umbrellas (and/or numbers), for each of the following:

Since each umbrella sells for a constant selling price of $19.99, we can immediately deduce that MR is always 19.99, and that TR is linear with slope 19.99 (TR=(price) x (quantity)=19.99q).

Since each umbrella costs the same amount $9.50 to produce, we can immediately deduce that MC is constant at 9.50, and that TC is linear with slope 9.50 (TC=(cost) x (quantity)+FC=9.50q+250).

$MR(q) =$ ___**19.99**___

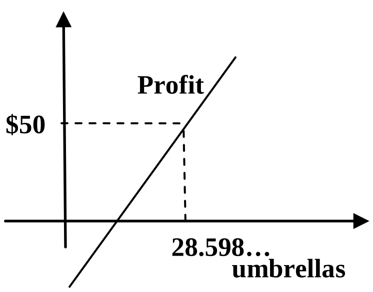
$MC(q) =$ ___**9.50**___

$TR(q) =$ ___**19.99*q***___

$TC(q) =$ ___**9.50*q*+250**___

b) (4 points) What is the smallest number of umbrellas which you need to produce and sell in order to make at least $50 in profit?

Profit ≥50, i.e. TR-TC=(19.99q)-(9.50q+250)=10.49q-250 ≥50. Solving for q (linear equation):



$$10.49q - 250 = 50$$
$$10.49q = 300$$
$$q = \frac{300}{10.49} \approx 28.598 \ldots umbrellas$$

The profit is an increasing line, so it's larger than 50 for quantities larger than $q = 28.598 \ldots$*, i.e from 29 umbrellas on.*

ANSWER: $q =$ ___***29***___ Umbrellas
(*your answer should be a whole number of umbrellas*)

c) (4 points) At what quantity is your average cost $14.50 per umbrella?

We're told that $AC = 14.50$***. But*** $AC = \frac{TC}{q} = \frac{9.50q+250}{q}$.

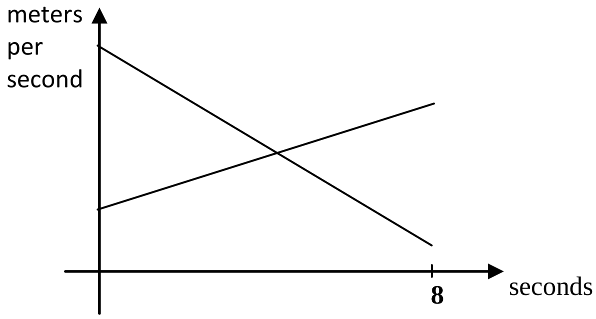
So we need to solve for q the equation: $\frac{9.50q+250}{q} = 14.50$***. Multiplying by q on both sides:***

$$9.50q + 250 = 14.50q$$
$$250 = 5q$$
$$q = \frac{250}{5} = 50$$

ANSWER: At $q =$ ___**50**___ Umbrellas

AUT 08, Nichifor, Midterm 2, v1 SOLUTIONS

2. Two toy cars are traveling on the same track, and both pass the start line at time $t = 0$.
The following functions and graphs are of the **instantaneous speeds** at t seconds for the two cars, during a 8-second trip.
This problem was based on WS 16



$s_A(t) = -1.4t + 12$ in meters per second

$s_B(t) = 0.6\,t + 3$ in meters per second

a) (3 points) What is the lowest instantaneous speed for car A during this 8 second trip?

The graph and formula for s_A are given: it is a decreasing line.
Its minimum will be at its farthest point (t=8), so lowest speed for car A is:

$$\boldsymbol{s_A(8) = -1.4(8) + 12 = 0.8}$$

ANSWER: ____**0.8**_______ meters per second

b) (4 points) When during this trip is the distance between the cars maximal? Which car is ahead at that time?

We saw in WS 16 that the distance between the cars is maximal when the speeds are equal.
So set $s_A(t) = s_B(t)$, and solve for t.

$$\boldsymbol{-1.4t + 12 = 0.6t + 3}$$
$$\boldsymbol{9 = 2t}$$
$$\boldsymbol{t = \frac{9}{2} = 4.5}$$

Car A is ahead at t=4.5 sec because they both start at the same place but A drives faster than B the entire time (see the graphs of speeds above)

ANSWER: The distance between the two cars is max at $t =$ __**4.5**__ seconds (car __***A***__ is ahead)

c) (4 points) Write down the distance *vs*. time functions for each car, and find the maximal distance between the two cars during this trip.

$D_A(t) =$ ___$\boldsymbol{-0.7t^2 + 12t}$_________

$D_B(t) =$ ______$\boldsymbol{0.3t^2 + 3t}$_________

(use the formulas from WS16)

In part (b) we saw that the distance is max at *t=4.5* and car A is ahead. So, compute

$$\boldsymbol{D_A(4.5) - D_B(4.5) = \cdots = 20.25}$$

ANSWER: max distance between the cars is ________20.25________ meters

AUT 08, Nichifor, Midterm 2, v1 SOLUTIONS

3. You produce Widgets. Your Total Revenue and Variable Cost are given by the following functions:

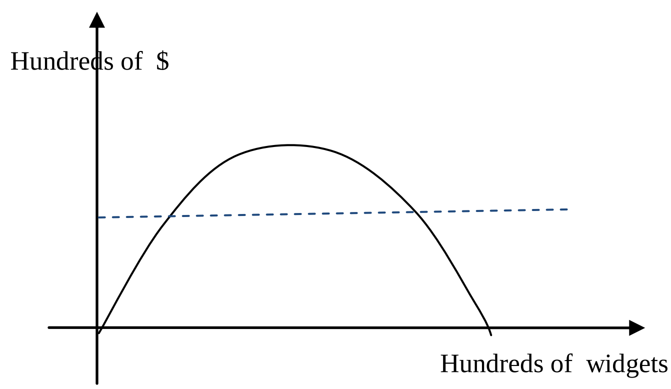
$$TR(q) = -0.5q^2 + 6q$$

$$VC(q) = 0.01q^3 - 0.2q^2 + 1.5q$$

with quantity q in **hundreds of Widgets**, and the total revenue and variable cost in **hundreds of dollars**.

This pbl was based on WS15

a) (5 points) Sketch the Total Revenue graph and find the maximum Total Revenue.



TR's graph is a concave-down parabola, so TR is max at its vertex:

$$q = -\frac{6}{2(-0.5)} = 6$$

Max value of TR is:

$$TR(6) = -0.5(6)^2 + 6(6) = 18$$

ANSWER: Maximum TR is ___**18**_________ hundred dollars

b) (5 points) Find **all** the quantities q for which the Total Revenue is above $1200.
Pay careful attention to units!

TR is in hundreds, so $1200=12 hundred dollars. Set TR=12 and solve for q:

$$-0.5q^2 + 6q = 12$$

$$-0.5q^2 + 6q - 12 = 0$$

Quadratic Formula: $q \approx 2.5358\ldots$ and $q \approx 9.4641\ldots$which corresponds to: 253.58 to 946.41 widgets

Drawing the dotted line at height 12 on above graph, we see that TR is above 12 between these values.

ANSWER: From ____*254*______ to _____*946*_______ **Widgets**
(your answer should be a range of whole numbers of Widgets)

c) (5 points) Recall that the shutdown price can be computed as the lowest value of the average variable cost. Compute the shutdown price. Include correct units.

$$\mathbf{AVC = \frac{VC}{q} = \frac{0.01q^3 - 0.2q^2 + 1.5q}{q} = 0.01q^2 - 0.2q + 1.5}$$

Graph of AVC is a concave-up parabola so its lowest point is at its vertex: $q = -\frac{-0.2}{2(0.01)} = 10$ widgets

$$\mathbf{SDP = AVC(10) = 0.01(10)^2 - 0.2(10) + 1.5 = \$0.5}$$

ANSWER: $SDP =$ ___**0.5**___ Units: ____$ (or $ per Widget)____

4. The distance, in miles, that a truck travels from its starting place in t minutes is given by the formula:

$$D(t) = t - 0.001\,t^2$$

This problem was based on the skills and problems from WS 11 and 13

a) (4 points) How long did the truck take to travel the first 20 miles? ***(like WS 13, pbl 22(a))***

Set the distance equal to20 miles and solve for t:

$$\boldsymbol{t - 0.001\,t^2 = 20}$$

$$\boldsymbol{-0.001\,t^2 + t - 20 = 0}$$

Quadratic Formula gives: t=20.416.., and t=979.58…. Pick the earlier time.

ANSWER: ______**20.42**________ minutes

b) (5 points) Find a formula in terms of h for the **average speed** of this truck **from $t = 30$ to $t = 30 + h$ minutes**. Simplify your formula as much as possible. ? (**like WS 11)**

$$\boldsymbol{AS = \frac{D(30+h) - D(30)}{\cancel{30} + h - \cancel{30}}}$$

$$\boldsymbol{D(30+h) = (30+h) - 0.001(30+h)^2 = 30 + h - 0.001\left(900 + 60h + h^2\right)}$$
$$\boldsymbol{= 30 + h - 0.9 - 0.06h - 0.001h^2 = 29.1 + 0.94h - 0.001h^2}$$

D(30)=…=29.1

$$\boldsymbol{AS = \frac{D(30+h) - D(30)}{h} = \frac{\left(\cancel{29.1} + 0.94h - 0.001h^2\right) - (\cancel{29.1})}{h} =}$$
$$\boldsymbol{= \frac{0.94h - 0.001h^2}{h} = 0.94 - 0.001h}$$

ANSWER: $AS(h) =$ ____ $\boldsymbol{0.94 - 0.001h}$ __________

c) (3 points) Another vehicle, a van, travels on the same road and in the same direction at a **constant speed** of 0.9 miles per minute, and reaches the starting place of the truck at $t = 5$ minutes. Find the distance *vs.* time formula for the van, $V(t)$.

Constant speed means the distance is linear, with slope =speed.

So the van's distance is of the form $V(t) = 0.9t + b$.

The van is at the starting place of the truck (distance=0) at t=5. That is, $V(5) = 0$:

$$\boldsymbol{0.9(5) + b = 0}$$

$$\boldsymbol{b = -0.9(5) = -4.5}$$

ANSWER: $V(t) =$_____ $\boldsymbol{0.9t - 4.5}$_______________